Fluid Mechanics
Unit 2- Fluid Statics

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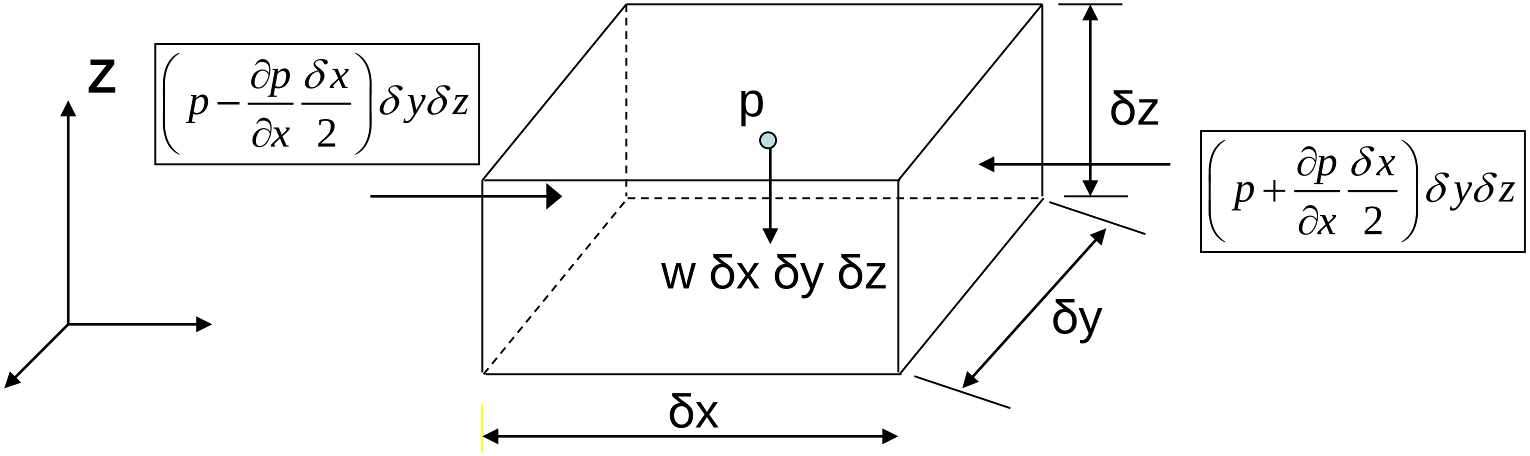
Fluid Statics

- **Fluid Pressure or pressure intensity,** $p = dF / dA \qquad p \neq F / A$
- **Force always acts in the direction normal to the wall**
- **SI unit – N/m^2 or Pascal**

Variation of Pressure in a Fluid



For equilibrium $\sum F_x = 0 \quad \sum F_z = 0 \quad \sum F_z = 0$

Fluid Pressure and Its Measurement

$$\sum F_X = 0 \Rightarrow \frac{\partial p}{\partial x} = 0 \qquad \sum F_Y = 0 \Rightarrow \frac{\partial p}{\partial y} = 0$$

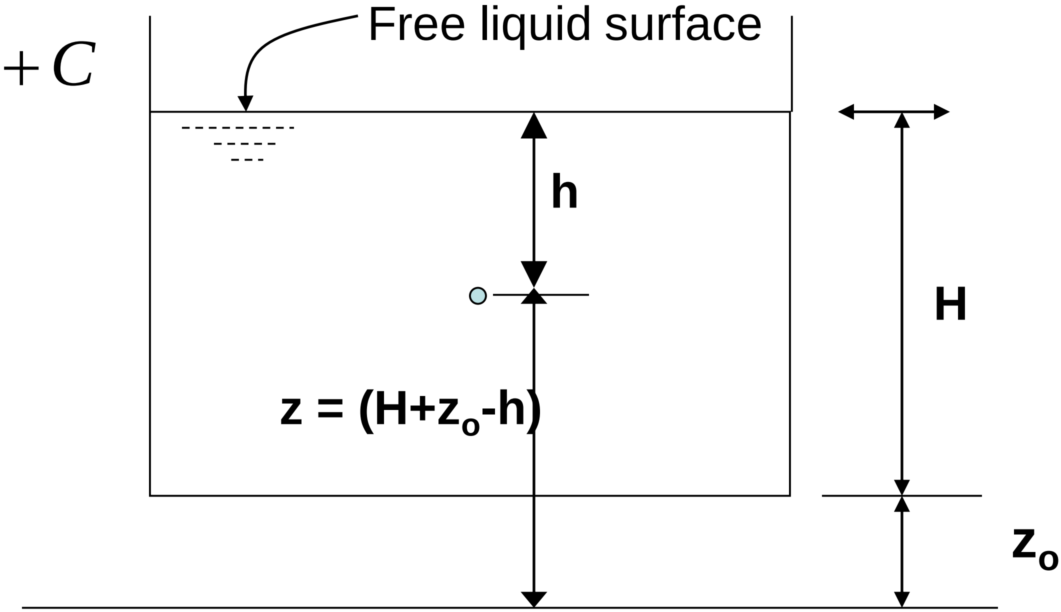
$$\sum F_Z = 0 \Rightarrow \frac{\partial p}{\partial z} = -w$$

$$\frac{dp}{dz} = -w = \rho g$$

Thus the pressure intensity varies only in the vertical direction in the static mass of fluid and it increases with the depth of the fluid

Pressure at a point in a liquid

We know $\frac{dp}{dz} = -w \Rightarrow p = -wz + C$



- **For free surface pressure is atmospheric, p_a**
- **z = (H+z_o)**
- **Hence C = [pa +w(H+z_o)**

$$\Downarrow$$

$$p = -wz + [p_a + w(H + z_o)]$$

For point in liquid $z = (H + z_o - h)$ **Thus,** $p = p_a + wh$

If atmospheric pressure is considered as datum $p = wh$

$$p = w_1 h_1 = w_2 h_2 \Rightarrow S_1 h_1 = S_2 h_2$$

Pressure head

- The vertical height of the free surface above any point in a liquid at rest is known as pressure head. Thus,

$$h = \frac{p}{w}$$

- It is a convenient to express the pressure in terms of vertical height of the liquid since the pressure depends on only vertical height

- Thus pressure can be expressed in meters of liquid column

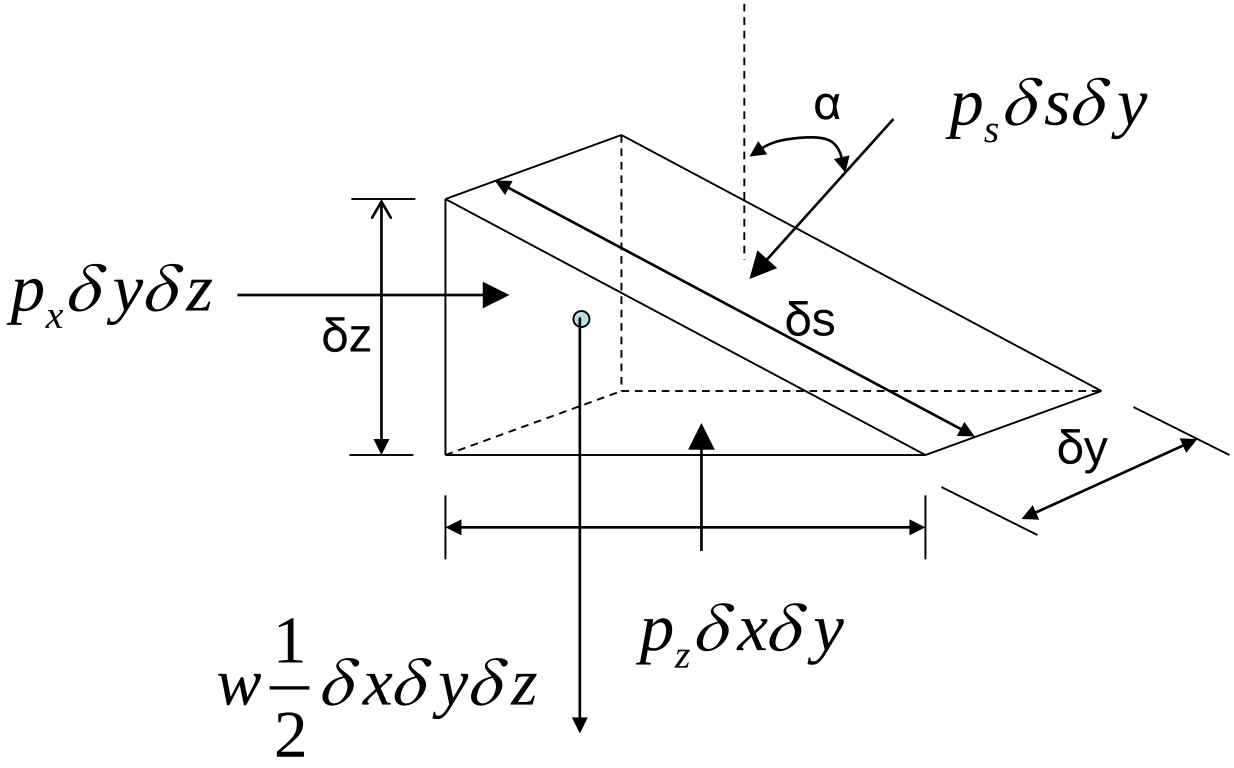
- The equation *p* = *wh* can be used to obtain the relationship between heights of columns of different liquids for same pressure

$$p = w_1 h_1 = w_2 h_2 \Rightarrow wS_1 h_1 = wS_2 h_2 \Rightarrow S_1 h_1 = S_2 h_2$$

Where, w is the specific weight of water and S1 and S2 are the specific gravities of two different fluids

Pascal's Law

- ❑ **Pressure at a point in a fluid acts with same magnitude in all the directions**



Pascal's Law

- **Resolving the forces along X and Z direction and equating with zero gives:**

$$p_x\delta y\delta z - p_s\delta y\delta s\sin\alpha = 0$$

$$p_z\delta x\delta y - p_s\delta y\delta s\cos\alpha - w\frac{1}{2}\cancelto{0}{\delta x\delta y\delta z} = 0$$

$$\delta s\sin\alpha = \delta z \ \ \textbf{and} \ \ \delta s\cos\alpha = \delta x$$

Thus,

$$p_x - p_s = 0$$

$$p_z - p_s = 0$$

Hence

$$p_s = p_x = p_z$$

Which means pressure acts equally in all directions as S direction is arbitrarily chosen. This is known as Pascal's law

Atmospheric, absolute, gage and vacuum pressure

- Atmospheric air exerts normal pressure up on all the surfaces with which it is in contact and it is known as atmospheric pressure (Varies with altitude, measured by barometer and hence called barometric pressure)

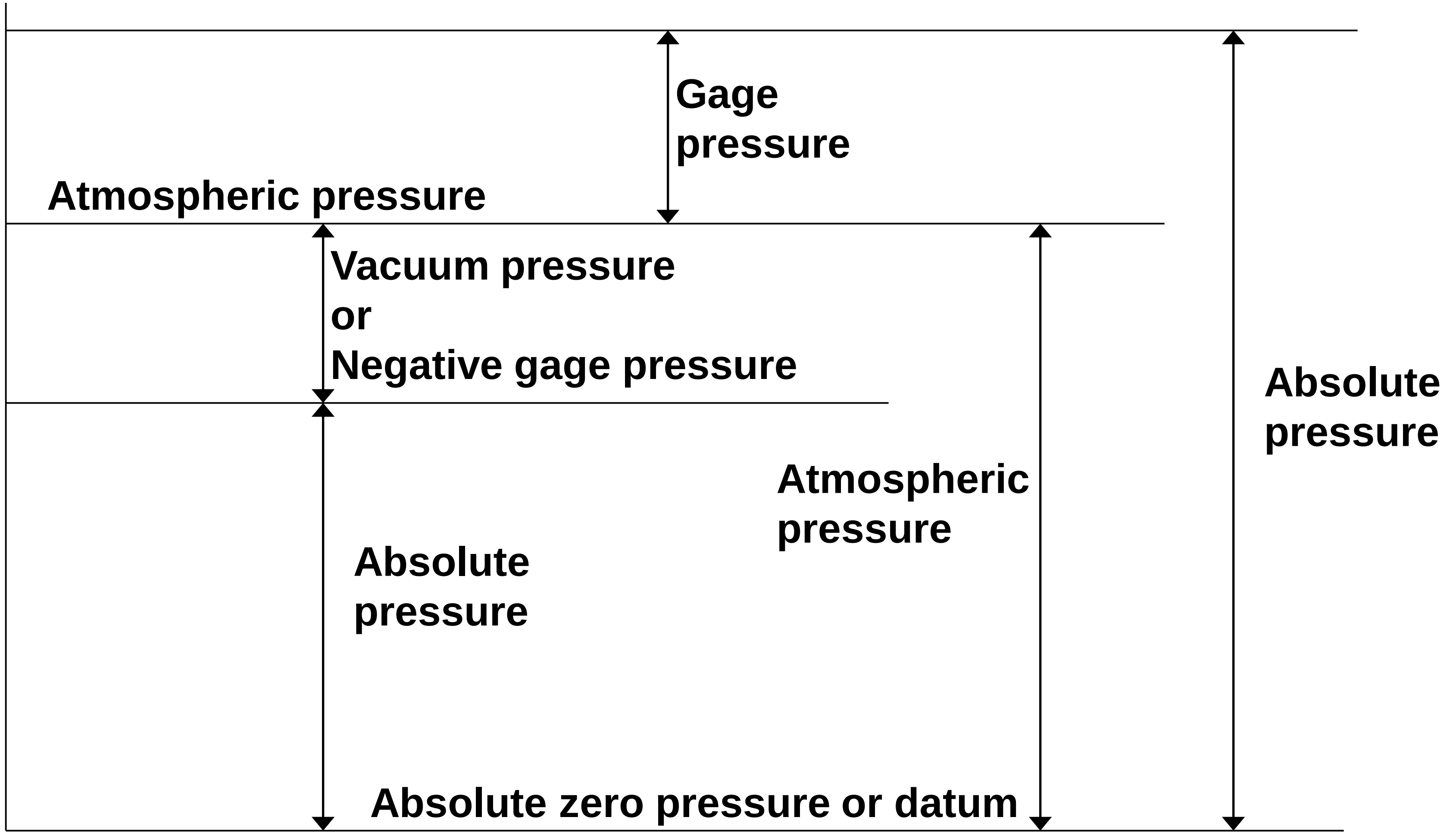
- At sea level – 10.1043×10^4 N/m^2 or 1.01043 bar or 10.3 m of water or 76 cm of Hg.

- Absolute zero or local atmospheric pressure can be the base for measurement

- If measured with reference to atm pressure it is called gage pressure (Negative gage pressure is vacuum)

- If measured with respect to absolute zero it is called absolute pressure

Atmospheric, absolute, gage and vacuum pressure



Pressure Measurement

Manometers

Mechanical gauges

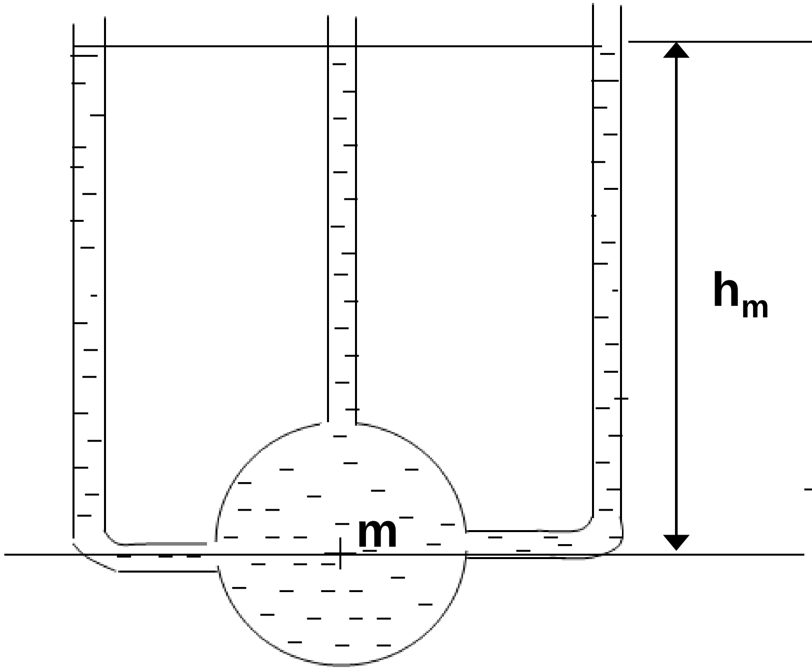
Simple Manometers

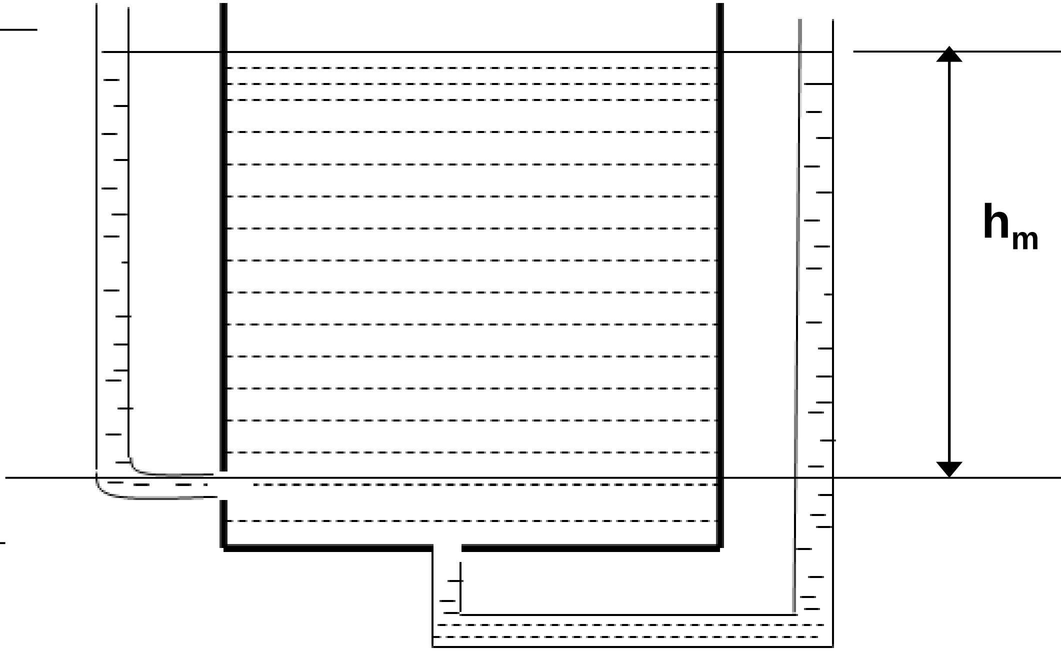
- ❑ Piezometers
- ❑ U-tube manometers
- ❑ Single column manometers
- ❑ Inclined single column manometers

Differential Manometers

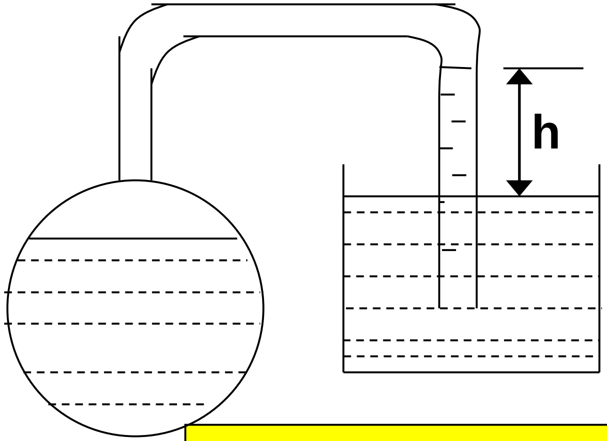
- ❑ Two piezometer manometer
- ❑ Inverted U-tube manometers
- ❑ U-Tube manometers
- ❑ Micro manometers

Piezometers –simplest manometers



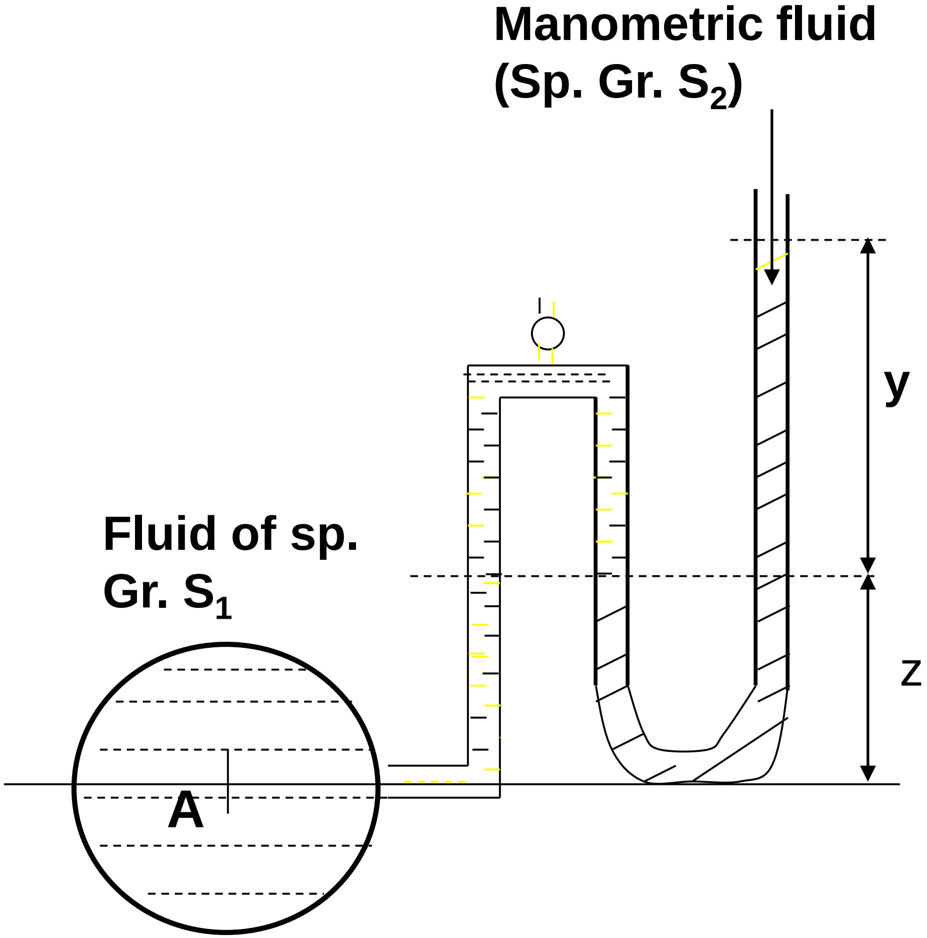


- Can measure only moderate pressures
- Location of insertion makes no difference
- Can not be used for gases (no free surfaces are formed)



Negative pressure measurement

U-Tube manometers



- A glass U-tube with heavier manometric fluid is used

Procedure to write manometric expression

- Start from A or free surface and write pressure there in appropriate unit (N/m^2 or m of H_2O)
- Add the change in pressure caused due to change from one level to adjacent level.
- Use +ve if the adjacent level is lower.
- Use –ve sign if it is higher.
- Continue till other end and equate with pressure at that point

U-Tube manometers

- **Manometric expression in terms of liquid at A**

$$\frac{P_A}{wS_1} - z - y\frac{S_2}{S_1} = 0$$

Where *w* is the specific weight of water

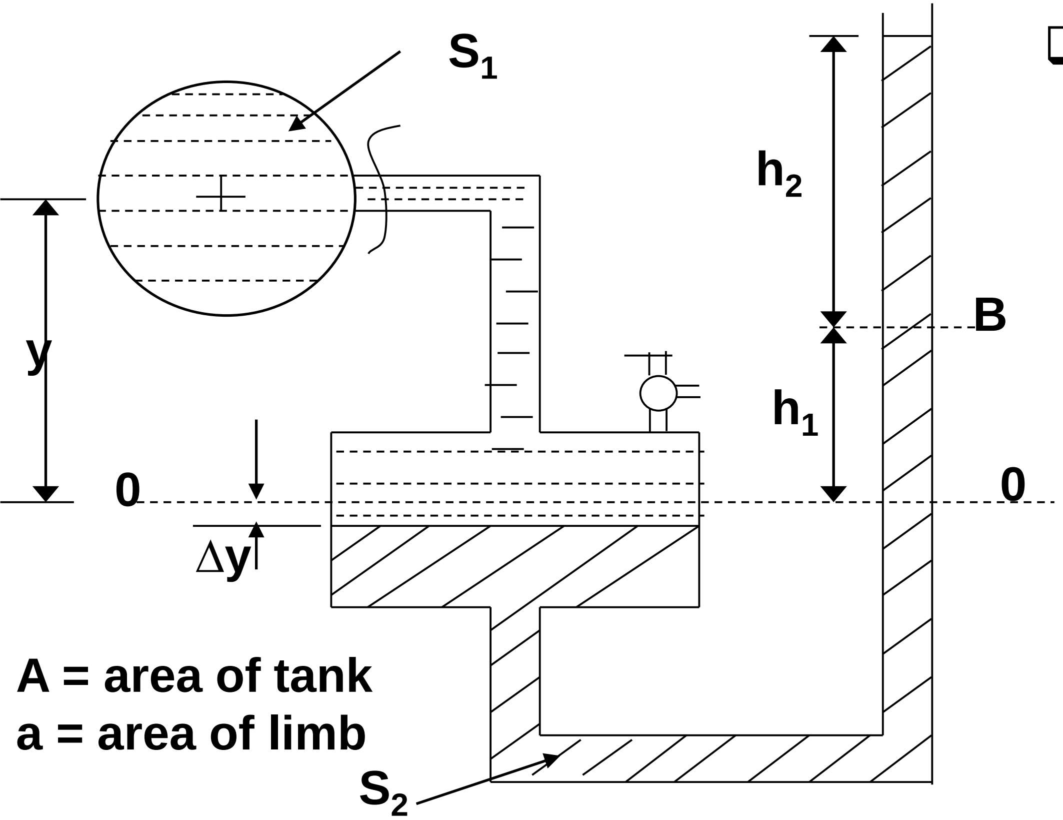
- **Manometric expression in terms of water**

$$\frac{P_A}{w} - zS_1 - yS_2 = 0$$

- **If A contains gas $S_1 = 0$**

$$\frac{P_A}{w} - yS_2 = 0$$

Single column manometers



- ☐ One of the limbs of U-tube manometer is replaced by a reservoir of large cross section

When not connected $yS_1 = h_1 S_2$

When connected $A\Delta y = a h_2$

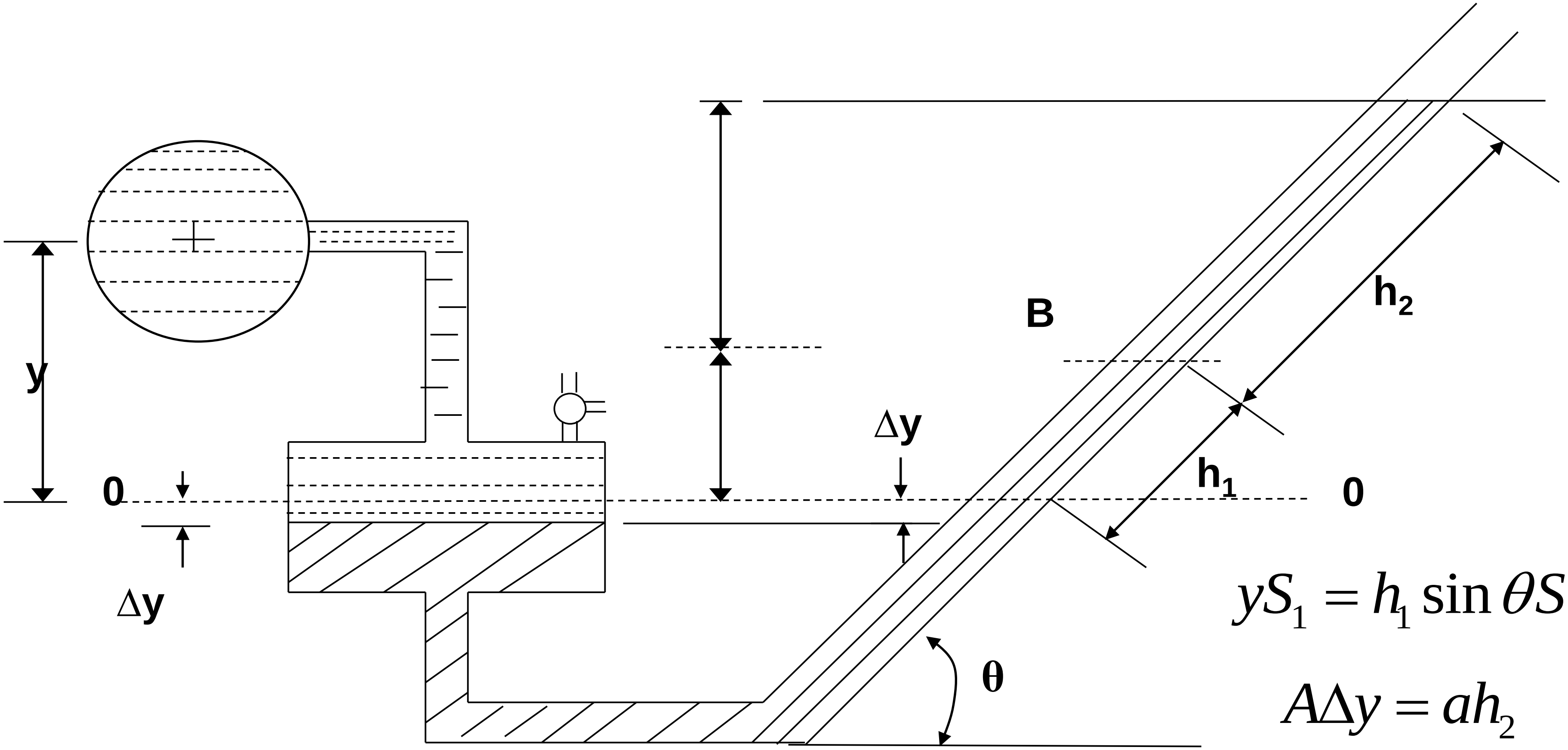
A = area of tank
a = area of limb

- ➢ The manometric expression starting from free end:

$$0 + (h_2 + h_1 + \Delta y)S_2 - (\Delta y + y)S_1 = \frac{P_A}{w}$$

$$\frac{P_A}{w} = h_2\left[(S_2 + (S_2 - S_1)\cancelto{0}{\frac{a}{A}}\right] \Rightarrow \boxed{\frac{P_A}{w} = h_2 S_2}$$

Inclined single column manometers



$$yS_1 = h_1 \sin\theta S_2$$

$$A\Delta y = ah_2$$

$$0 + (h_2 + h_1)\sin\theta S_2 + (\Delta y)S_2 - \Delta y S_1 - yS_1 = \frac{P_A}{w}$$

$$\frac{P_A}{w} = h_2\left[(S_2 \sin\theta + (S_2 - S_1)\frac{a}{A}\right] \Rightarrow \frac{P_A}{w} = (h_2 \sin\theta)S_2$$

Differential manometers

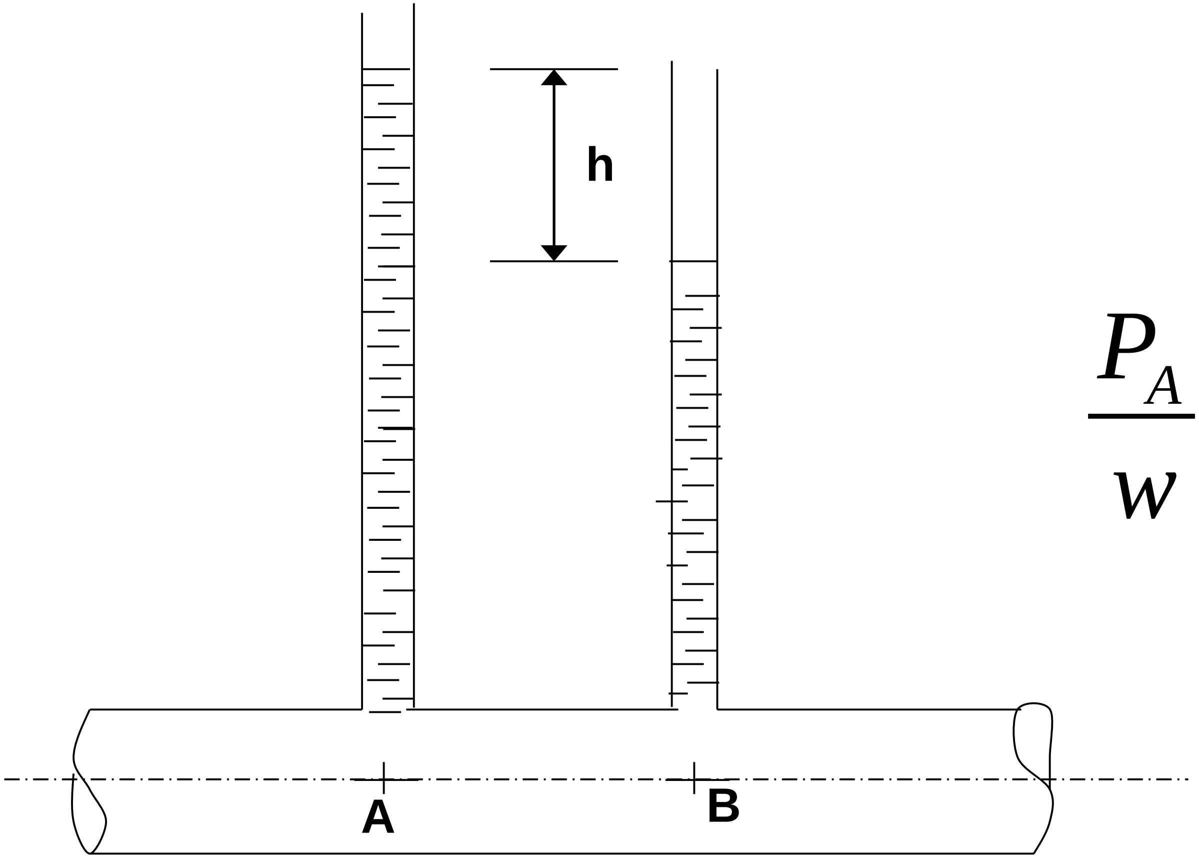
- Used for measuring the difference of pressure between any two points in a pipeline or in two pipes or a containers

- Usually consists of a glass U-tube two ends of which are connected to two gage points

Common types of differential manometers

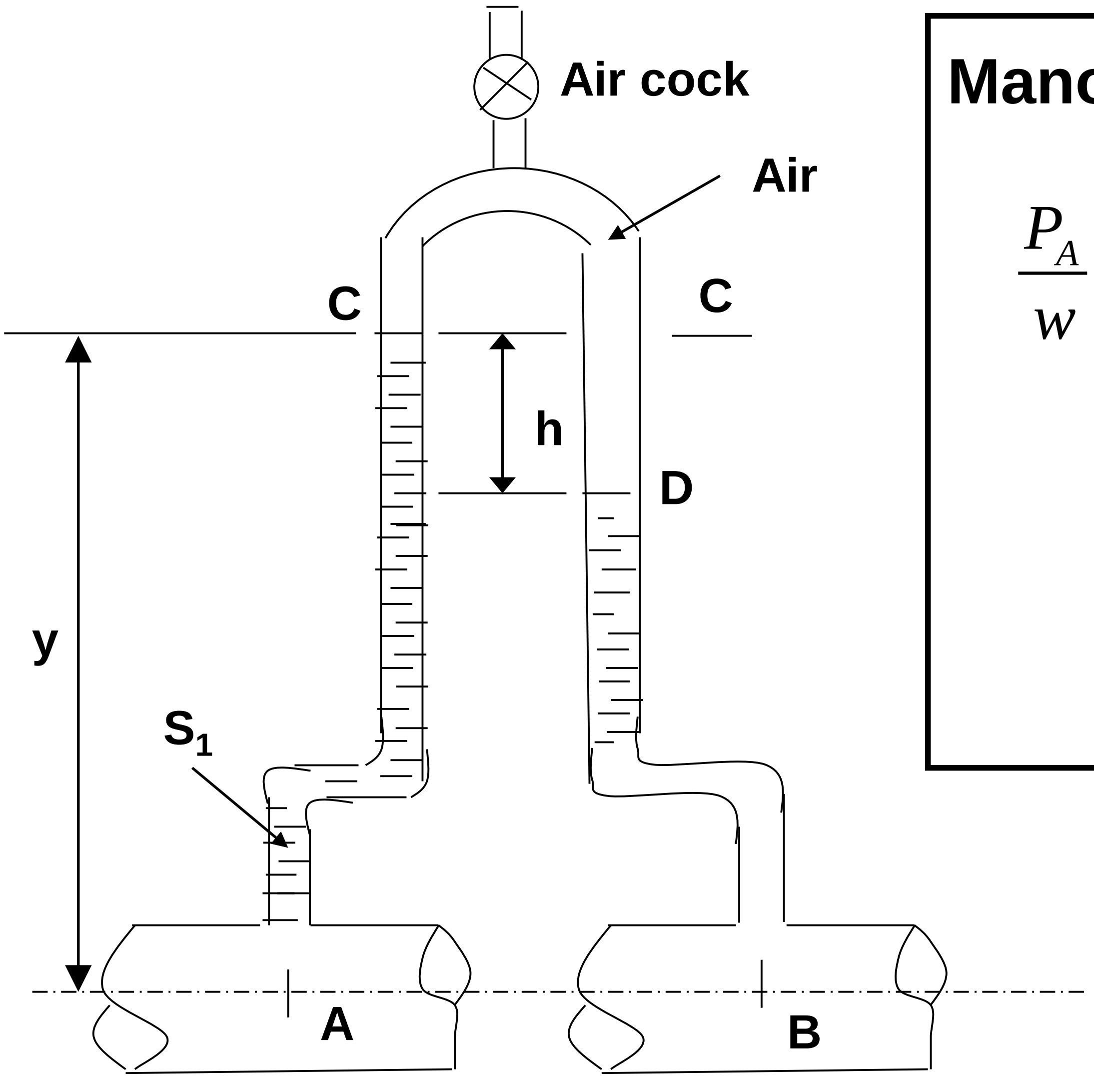
- Two piezometer manometer
- Inverted U-tube manometers
- U-Tube manometers
- Micro manometers

Two-Piezometer Differential Manometer



$$\frac{P_A}{w} - \frac{P_B}{w} = h$$

Inverted U-Tube Differential Manometer



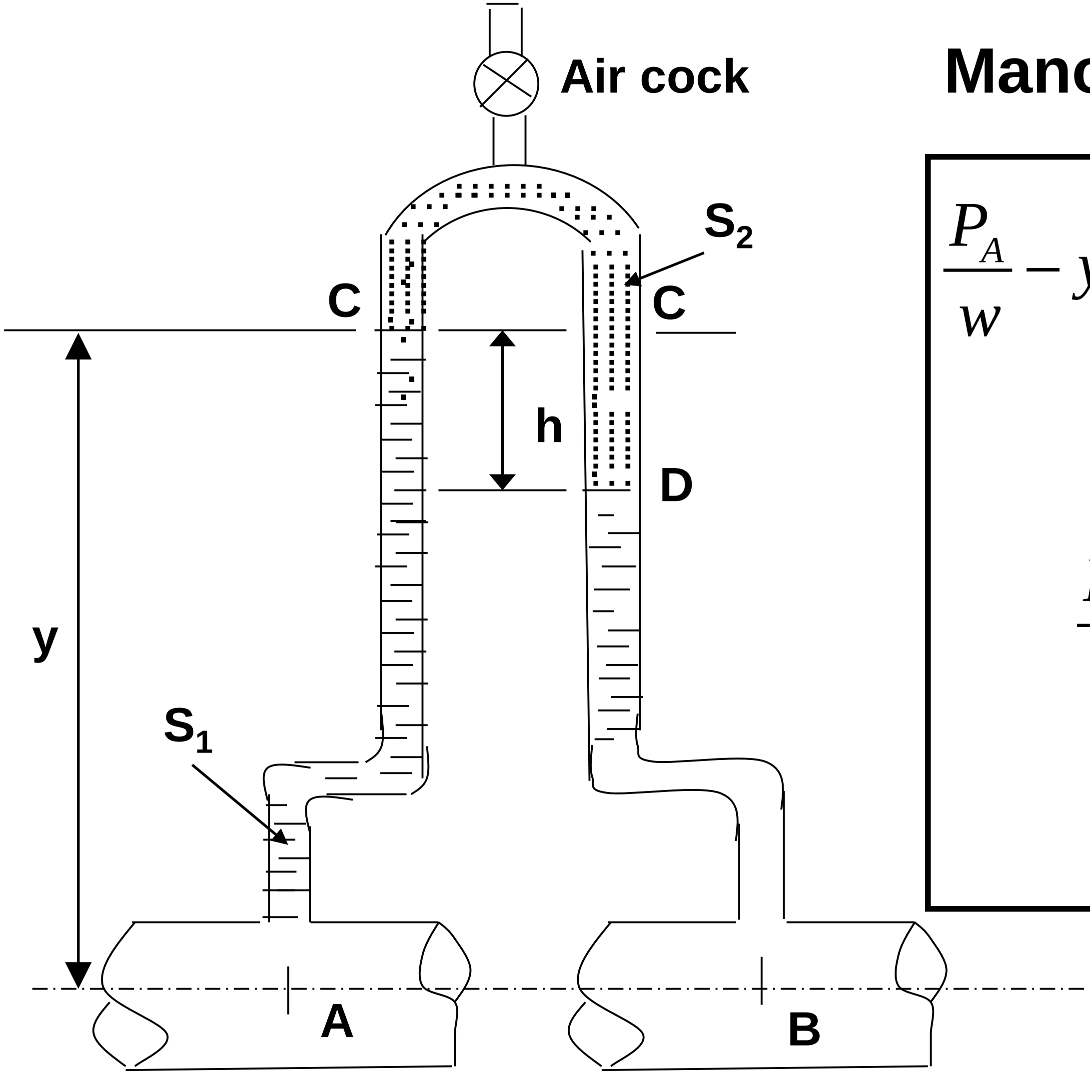
With air as manometric fluid

Manometric expression

$$\frac{P_A}{w} - yS_1 + (y-h)S_1 = \frac{P_B}{w}$$

$$\frac{P_A}{w} - \frac{P_B}{w} = hS_1$$

Inverted U-Tube Differential Manometer



With lighter fluid as manometric fluid

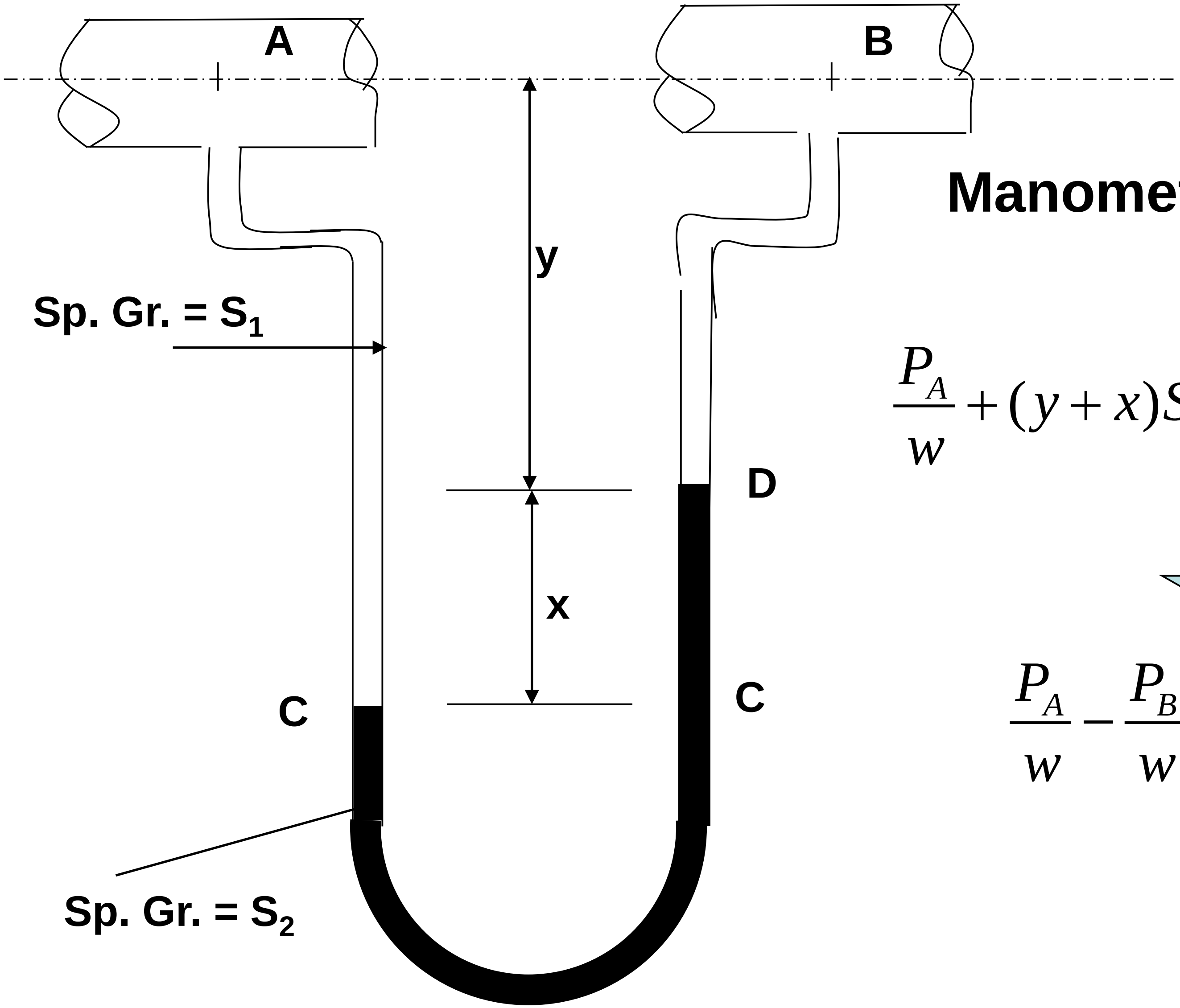
Manometric expression

$$\frac{P_A}{w} - yS_1 + hS_2 + (y-h)S_1 = \frac{P_B}{w}$$

⇩

$$\frac{P_A}{w} - \frac{P_B}{w} = h(S_1 - S_2)$$

U-Tube Differential Manometer

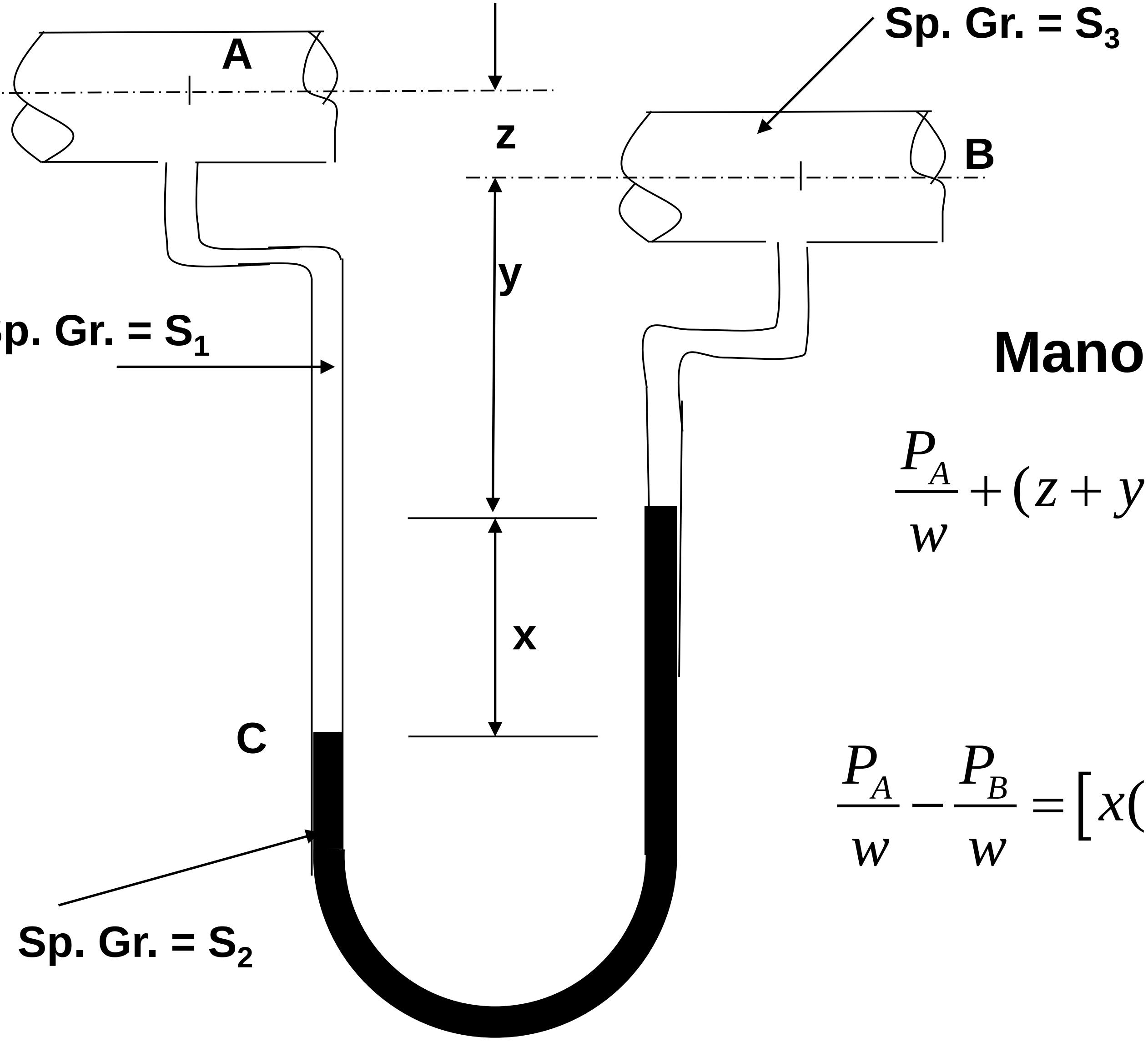


Manometric expression

$$\frac{P_A}{w} + (y + x)S_1 - xS_2 - yS_1 = \frac{P_B}{w}$$

$$\frac{P_A}{w} - \frac{P_B}{w} = x(S_2 - S_1)$$

U-Tube Differential Manometer with two points at different levels

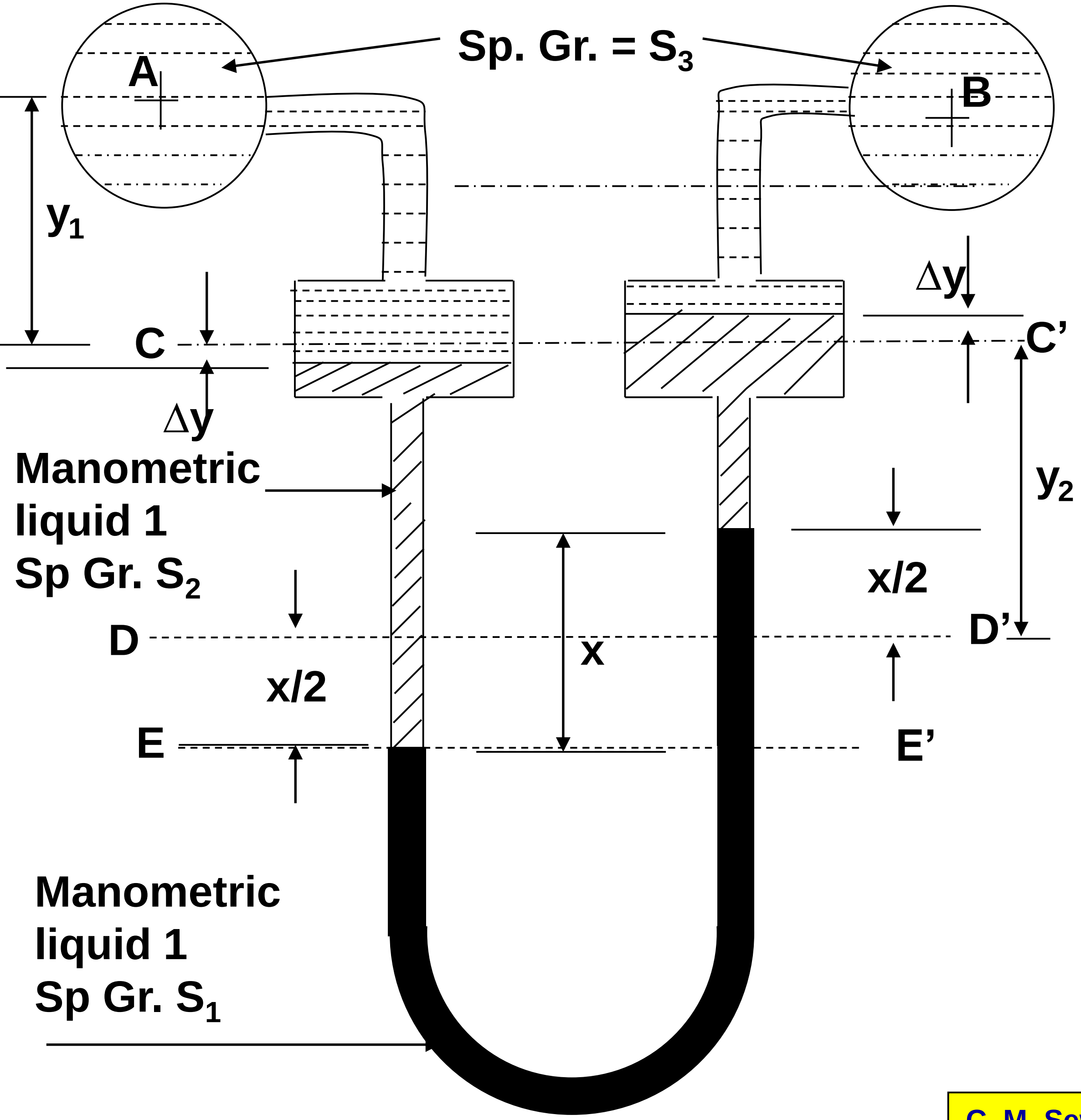


Manometric expression

$$\frac{P_A}{w}+(z+y+x)S_1-xS_2-yS_3=\frac{P_B}{w}$$

$$\frac{P_A}{w}-\frac{P_B}{w}=\left[x(S_2-S_1)+y(S_3-S_1)-zS_1\right]$$

Micromanometer



- Used for measurement of very small difference in pressure with precision
- Two manometric fluids and two basins with large cross sectional area

☐ When not connected fluids stands at C-C' and D-D'

Volume displacement in basin and limb is same

$$A(\Delta y) = a\left(\frac{x}{2}\right)$$

Micromanometer

Manometric expression

$$\frac{P_A}{w} + (y_1 + \Delta y)S_3 + \left(y_2 - \Delta y + \frac{x}{2} \right) S_2$$

$$-xS_1 - \left(y_2 - \frac{x}{2} + \Delta y \right) S_2 - (y_1 - \Delta y)S_3 = \frac{P_B}{w}$$

⇓ $$\Delta y = \frac{a}{A}\left(\frac{x}{2}\right)$$

$$\frac{P_A}{w} - \frac{P_B}{w} = -y_1S_3 - \frac{a}{A}\frac{x}{2}S_3 - y_2S_2 + \frac{a}{A}\frac{x}{2}S_2 - \frac{x}{2}S_2$$

$$+xS_1 + y_2S_2 - \frac{x}{2}S_2 + \frac{a}{A}\frac{x}{2}S_2 + y_1S_3 - \frac{a}{A}\frac{x}{2}S_3$$

Micromanometer

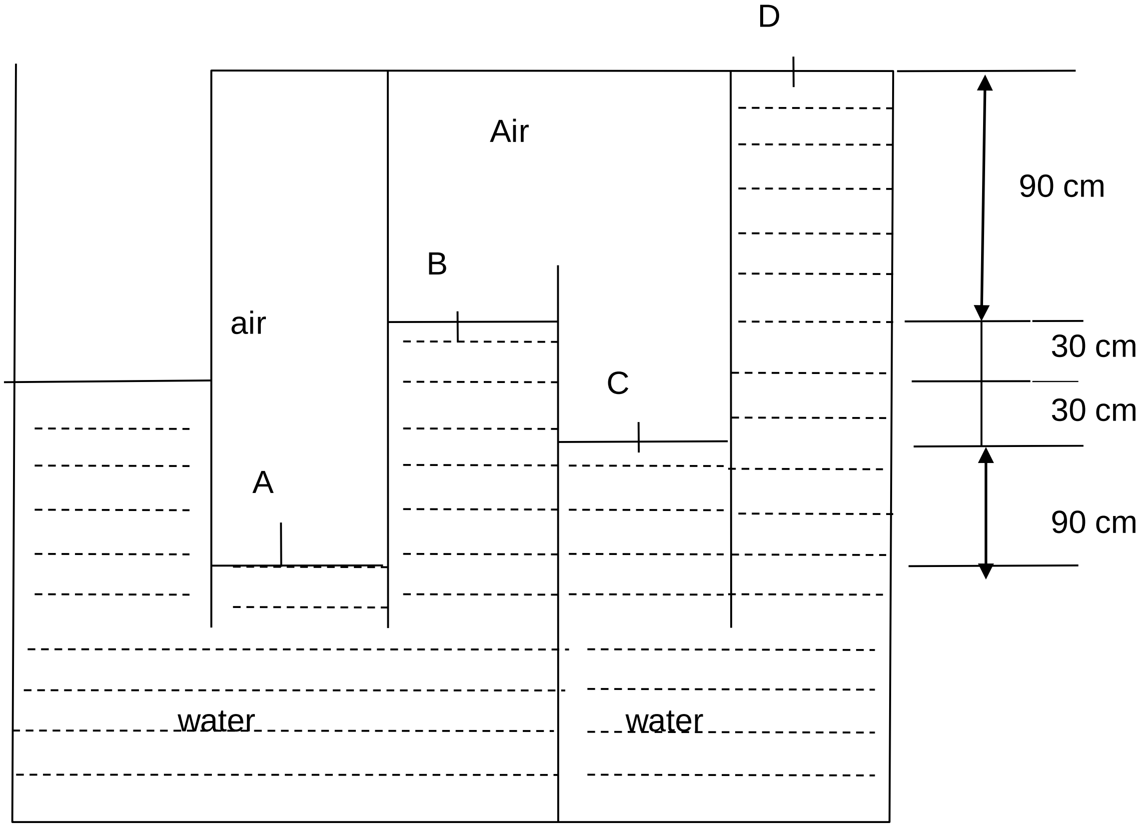
a<<A →

$$\frac{P_A}{w}-\frac{P_B}{w}=x\left[S_1-S_2\right]$$

- **Invented by Chattock Small and Krell**
- **These are very sensitive to pressure differences down to less than 0.0025 mm of water**
- **Disadvantage – appreciable time is required for taking the reading and hence are used for completely steady pressures**

What is the absolute pressure at a point 10 m below the free surface in a fluid that has a variable density in kilogram per cubic meters given by **ρ= 450+ah**, in which **a = 12 kg/m4** and **h** is the distance in meters measured from the free surface?

Determine the pressure at points A,B,C and D in Pascal’s



Hydrostatic Forces on Surfaces

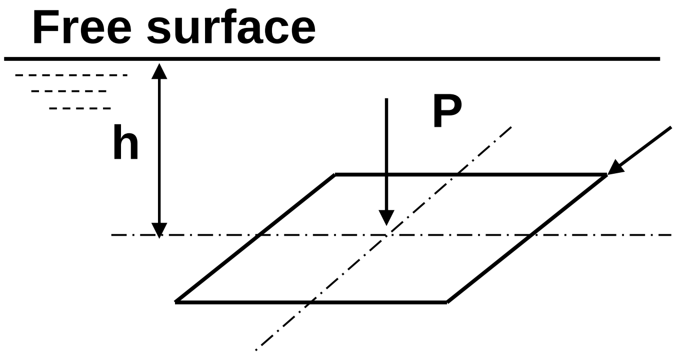
Important definitions

Total pressure – The force exerted by fluid on the surface which is immersed in the static mass of fluid is called total pressure. It is always exerted in the direction normal to the surface.

SI Unit - N

Centre of Pressure – Point of application of total pressure

Total Pressure on the horizontal surface

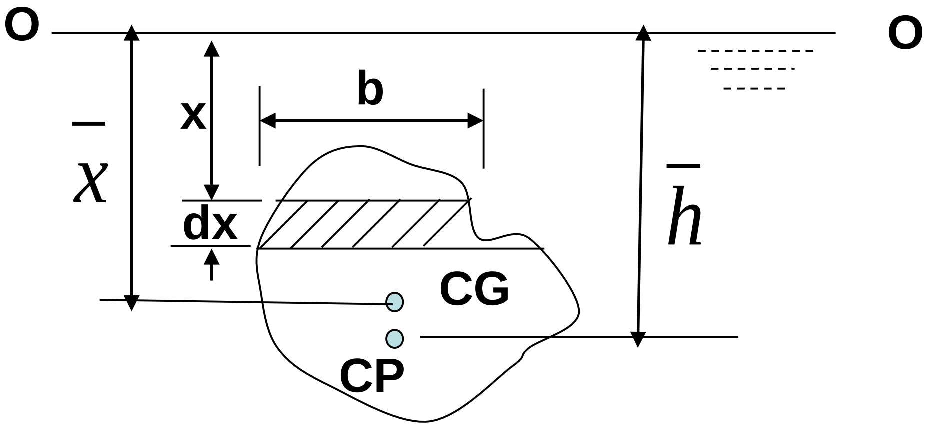


Horizontal plane of area A

$$P = pA = (wh)A = wAh$$

The centre of pressure is the centroid of the surface

Total Pressure on the Vertical Surface



- The pressure intensity is not constant on the surface since depth varies (Thus, $P \neq p \times A$)

- For horizontal strip of thickness ‘dx’ and width ‘b’

$$dA = b \times dx$$

- Total pressure on the strip $dP = wxdA = wxbdx$

Total Pressure on the Vertical Surface

- **Total pressure on entire plane** $P = \int dP = w \int x(bdx)$

- **The term $\int x(bdx)$ is the sum of first moment of areas of the strips about axis OO (through free surface)**

- **It is also given by the product of total area of the surface (A) and the distance of CG from the free surface (OO)**

Thus $\int x(bdx) = A\bar{x}$

$$P = w\bar{x}A$$

Hence the total pressure is equal to the product of pressure intensity at centroid and area of the surface

Centre of Pressure for a Vertical Surface

- **Since the pressure intensity varies with depth total pressure is not exerted through CG**

- **Moment of total pressure on the strip with OO**

$$dP \times x = wx(bdx)x = wx^2(bdx)$$

- **Sum of moment of total pressure with OO**

$$\int dP \times x = w\int x^2(bdx)$$

- **Moment of resultant of the system is equal to sum of the moments of components about the same axis**

$$P\bar{h} = w\int x^2(bdx)$$

Centre of Pressure for a Vertical Surface

- The term $\int x^2 (bdx)$ is the sum of the second moment of areas of the strips about axis OO (through free surface)

- It is also given moment of inertia about OO

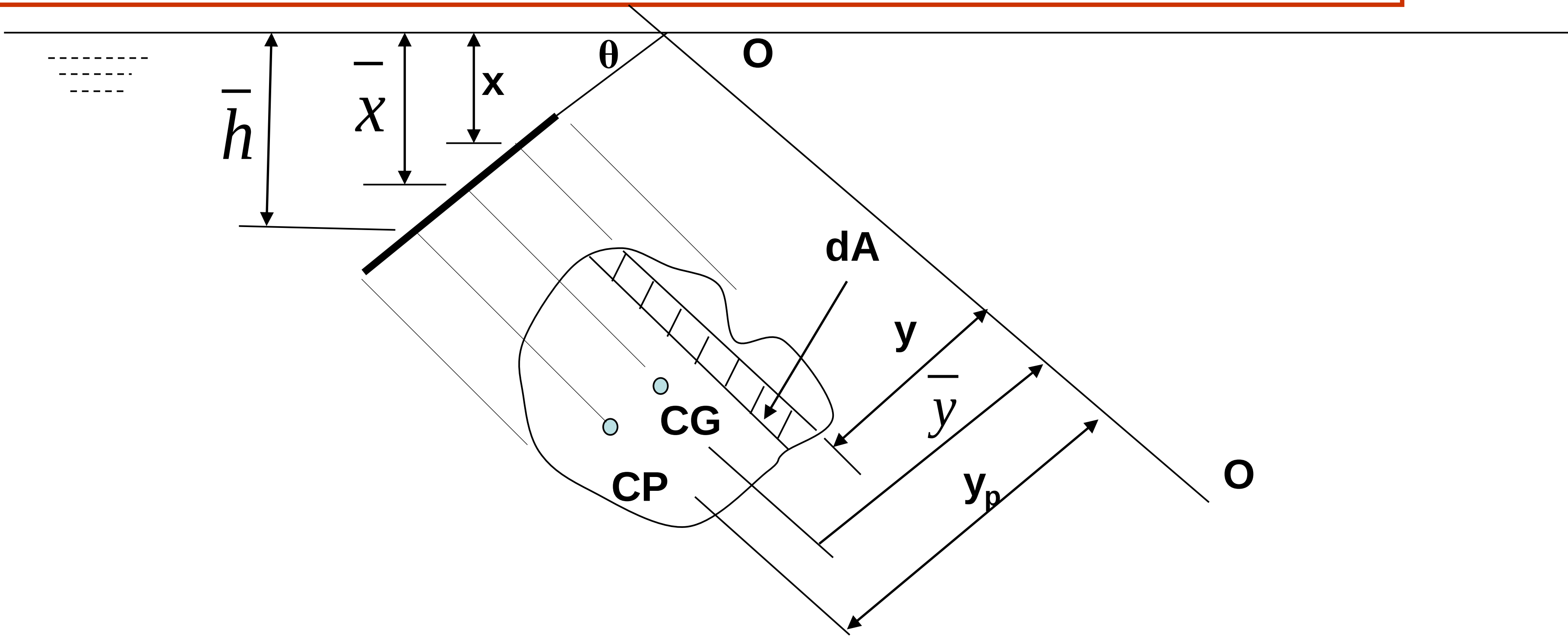
Thus $$I_O = \int x^2 (bdx)$$

$$P\bar{h} = wI_O \Longrightarrow \bar{h} = \frac{wI_O}{P} = \frac{wI_O}{wA\bar{x}}$$

We know $$I_O = I_G + A\bar{x}^2 \Longleftarrow$$ Parallel axis theorem

Thus $$\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}} \Longrightarrow \bar{h} > \bar{x}$$

Total Pressure on an Inclined Surface



- Total pressure on the strip $$dP = wxdA = w(y\sin\theta)dA$$

Thus $$P = \int ydA \times w\sin\theta$$

Total Pressure on an Inclined Surface

$\int y dA$ → **First moment of area given by** $A\bar{y}$

$$P = wA(\bar{y}\sin\theta)$$

$$P = w\bar{x}A$$

Hence the total pressure is equal to the product of pressure intensity at centroid and area of the surface which is also true for vertical surface

Centre of Pressure for a Inclined Surface

Let, $\bar{h}$ **= vertical depth of CP**

y_p **= distance of CP from OO along normal to free surface**

We know $$dP = w(y\sin\theta)dA$$

$$\Downarrow$$

$$dP \times y = w\sin\theta y^2 dA$$

Moment of total pressure with OO $= Py_p$

$$\Downarrow$$

$$Py_p = w\sin\theta \int y^2 dA$$

$$y_p = \frac{w\sin\theta I_O}{P} = \frac{w\sin\theta I_O}{wA\bar{x}}$$

$$\boxed{\int y^2 dA = I_O}$$

Centre of Pressure for a Inclined Surface

$$\boxed{I_O = I_G + A\bar{y}^2}$$

$$\boxed{y_p = \frac{\bar{h}}{\sin\theta}}$$

$$\boxed{\bar{y} = \frac{\bar{x}}{\sin\theta}}$$

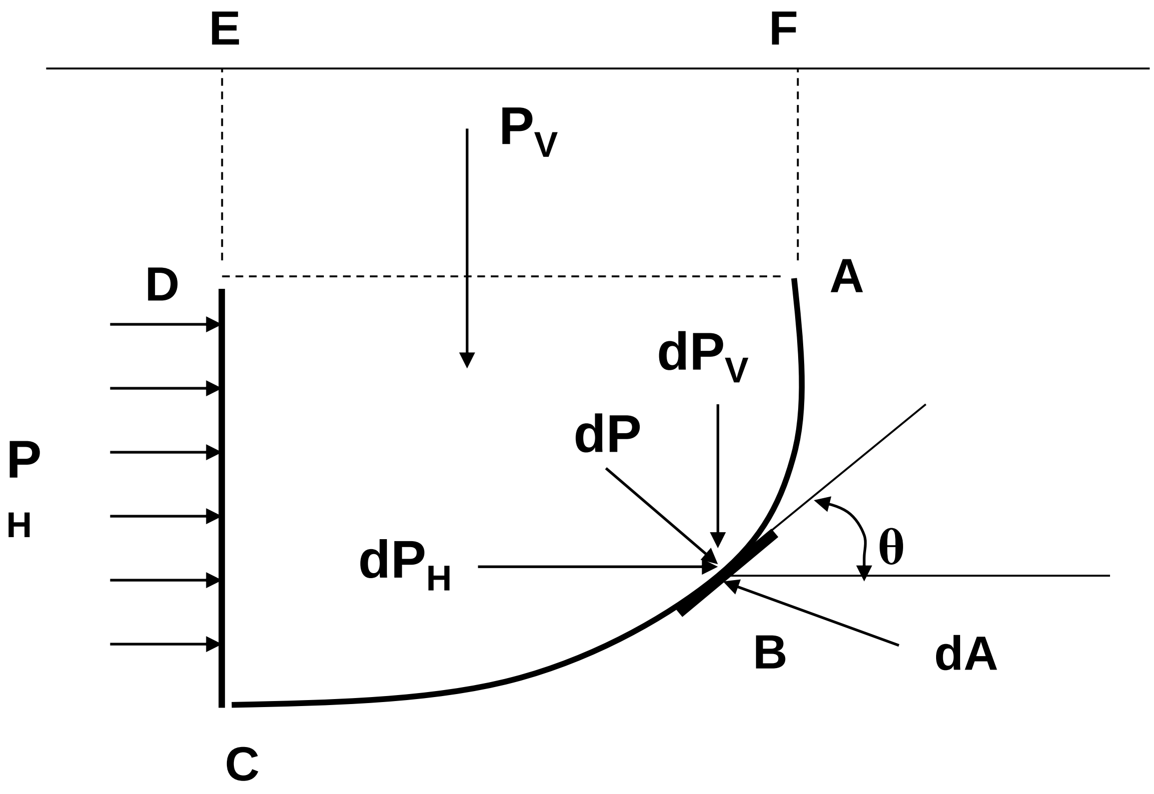
$$\bar{h} = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}$$

For θ = 90°

$$\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}}$$

Similar to vertical plane

Centre of Pressure for a Curved Surface



Direction of total pressure on the area of surface varies

$$P \neq \int p dA$$

The total pressure dP acting on dA can be resolved as

$$dP_H = dP \sin\theta = pdA \sin\theta$$

$$dP_V = dP \cos\theta = pdA \cos\theta$$

Centre of Pressure for a Curved Surface

$$P_H = \int dP_H = w\int h dA \sin\theta$$

$$P_V = \int dP_V = w\int h dA \cos\theta$$

- $dA\sin\theta$ **represents vertical projection of dA**
- $(wh)dA\sin\theta$ **represents total pressure on vertical trace of dA**
- **Thus component of total pressure in horizontal direction is total pressure on its vertically projected area CD**

Centre of Pressure for a Curved Surface

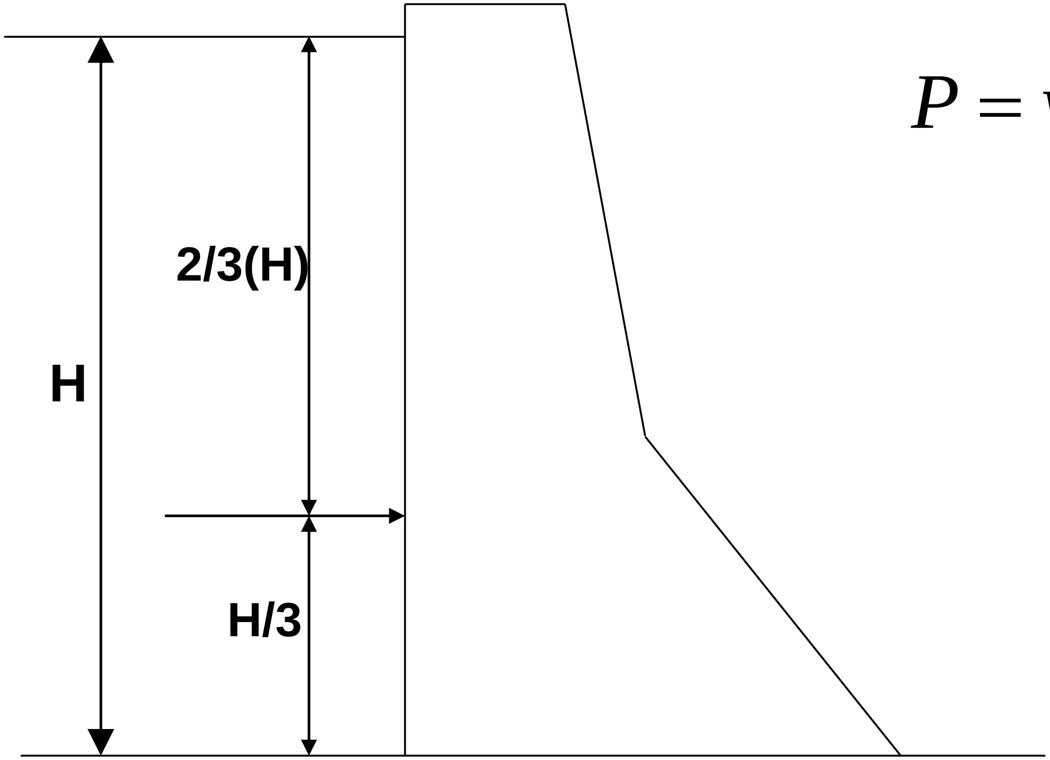
Similarly

- $dA\cos\theta$ **represents horizontal projection of dA**
- $(wh)dA\cos\theta$ **represents total pressure on horizontal trace of dA**
- **Thus component total pressure in the vertical direction is total pressure on its horizontally projected area**

$P_V = \int whdA\cos\theta$ **represents the weight of the liquid above ABCDEFA**

Thus, $P = \sqrt{P_H^2 + P_V^2}$ **and** $\theta = \tan^{-1}\left(\frac{P_V}{P_H}\right)$

Practical applications - Dams



Total Pressure per unit length of dam,

$$P = wA\bar{x} = w(H \times 1)\frac{H}{2}$$

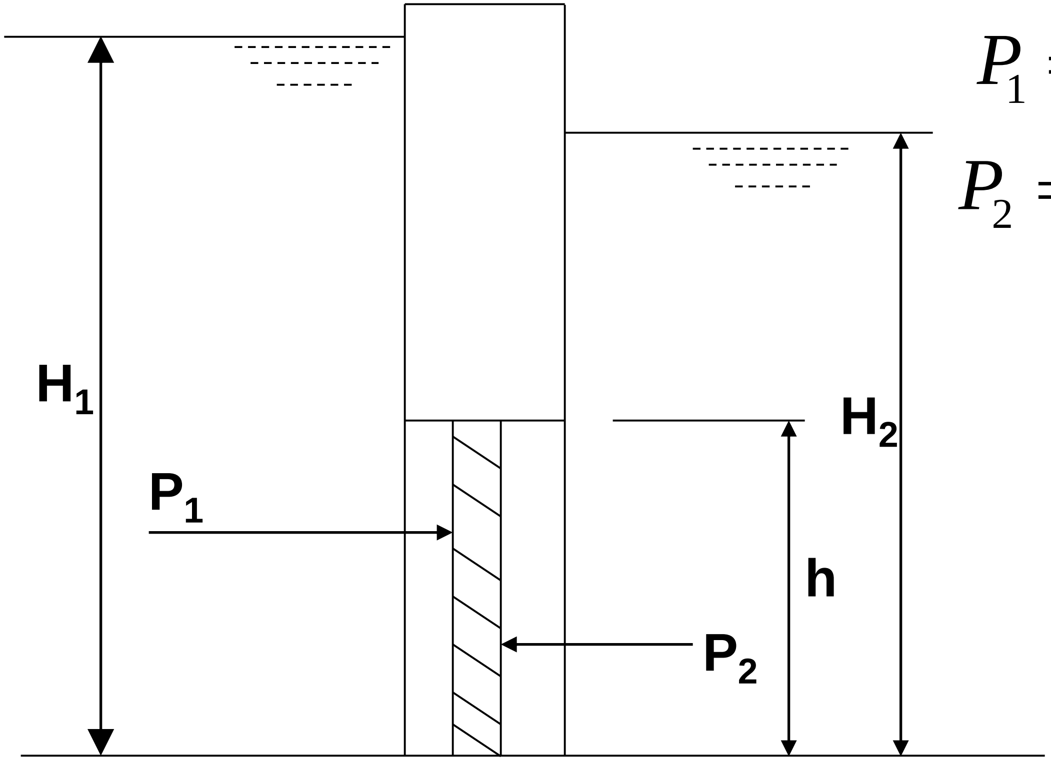
Centre of Pressure

$$\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}}$$

$$\bar{h} = \frac{H}{2} + \frac{\frac{1}{12} \times 1 \times H^3}{(H \times 1) \times H / 2}$$

$$\bar{h} = \frac{2}{3}H$$

Practical applications - Gates



Total Pressure per unit length of dam,

$$P_1 = wA\bar{x}_1 = wA(H_1 - h/2)$$

$$P_2 = wA\bar{x}_2 = wA(H_2 - h/2)$$

Resultant force experienced by the gate

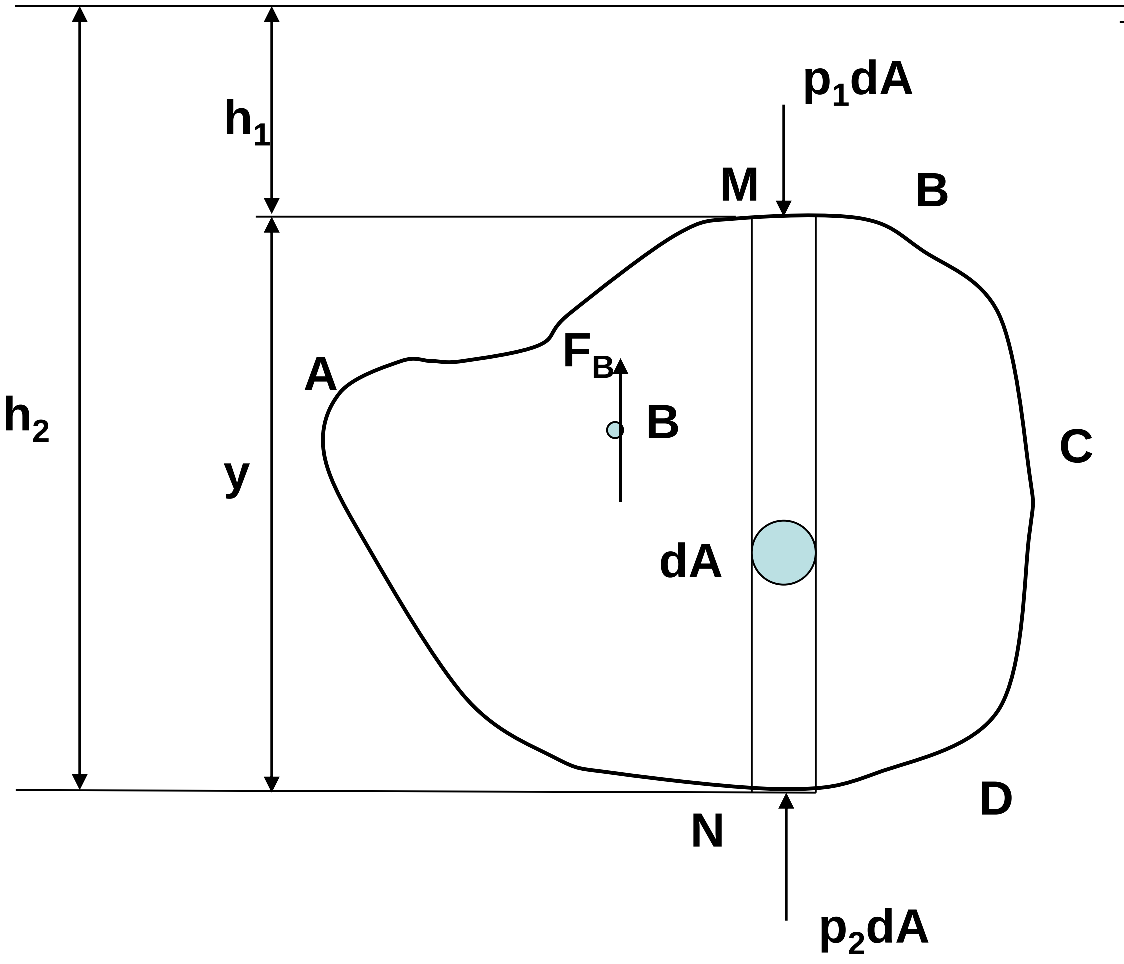
$$P = P_1 - P_2$$

Buoyancy and Floatation

Important definitions

- **Buoyancy** – Tendency of the partially or fully immersed body to be lifted up in the fluid is known as buoyancy.
- **Buoyant Force** – The force tending to lift the body upward is known as buoyant force
- **Center of Buoyancy** – The point of application of the buoyant force is known as center of buoyancy.
- **Archimedes' Principle** – When a body is fully or partially immersed in the fluid it is buoyed up by a force which is equal to the weight of fluid displaced by the body.

Buoyant Force on Fully Submerged Body



Specific weight = w

- Resultant horizontal force on the body = 0
- Buoyant force on the strip of elemental area dA

$$dF_B = (p_2 dA - p_1 dA)$$

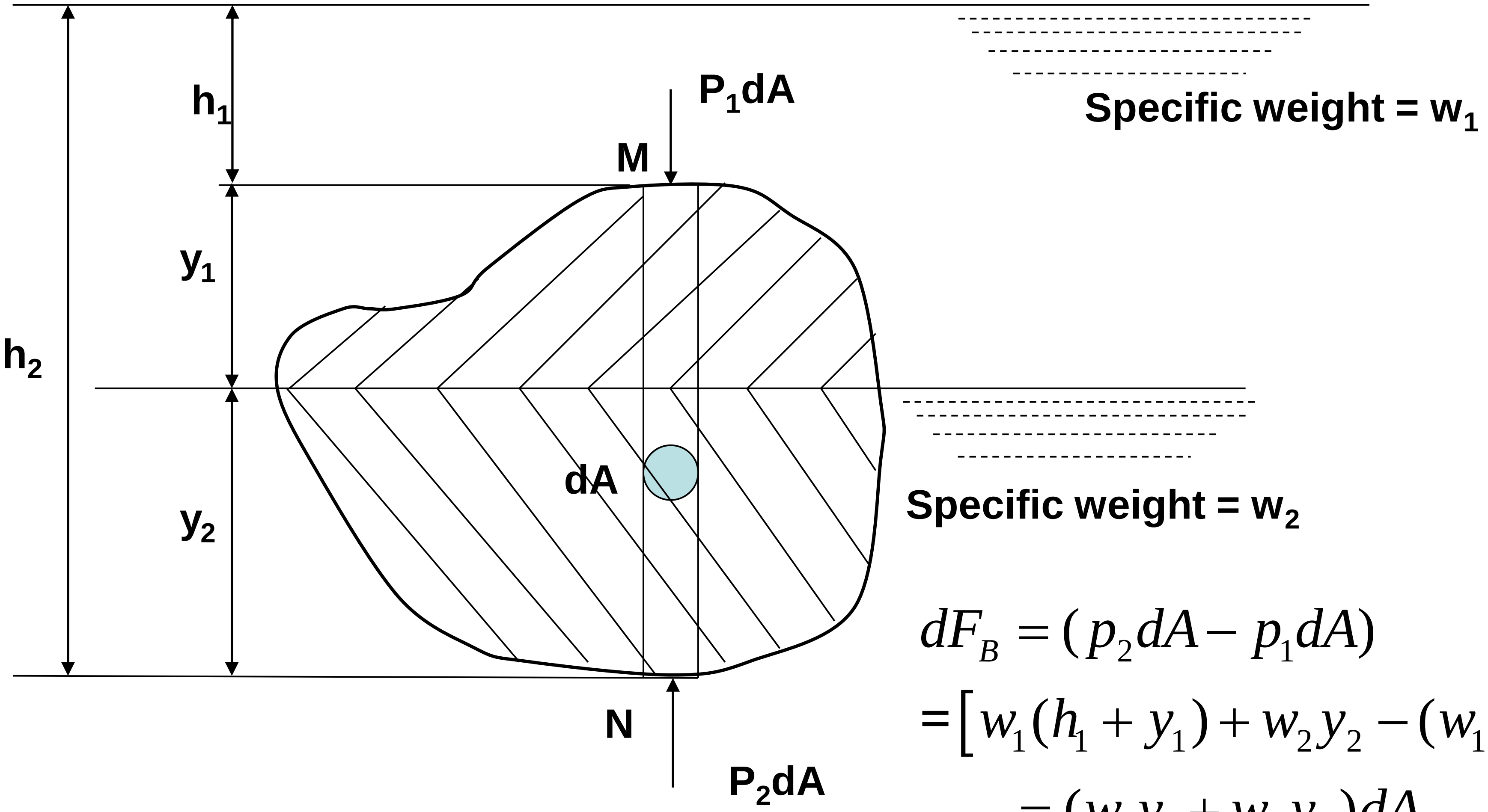
$$= w(h_2 - h_1)dA$$

$$= wydA$$

$$= wd\overline{V}$$

$$F_B = \int dF_B = \int wd\overline{V} = w\overline{V}$$

Buoyant Force on Partially Submerged Body



$$dF_B = (p_2 dA - p_1 dA)$$

$$= \left[w_1(h_1 + y_1) + w_2 y_2 - (w_1 h_1) \right] dA$$

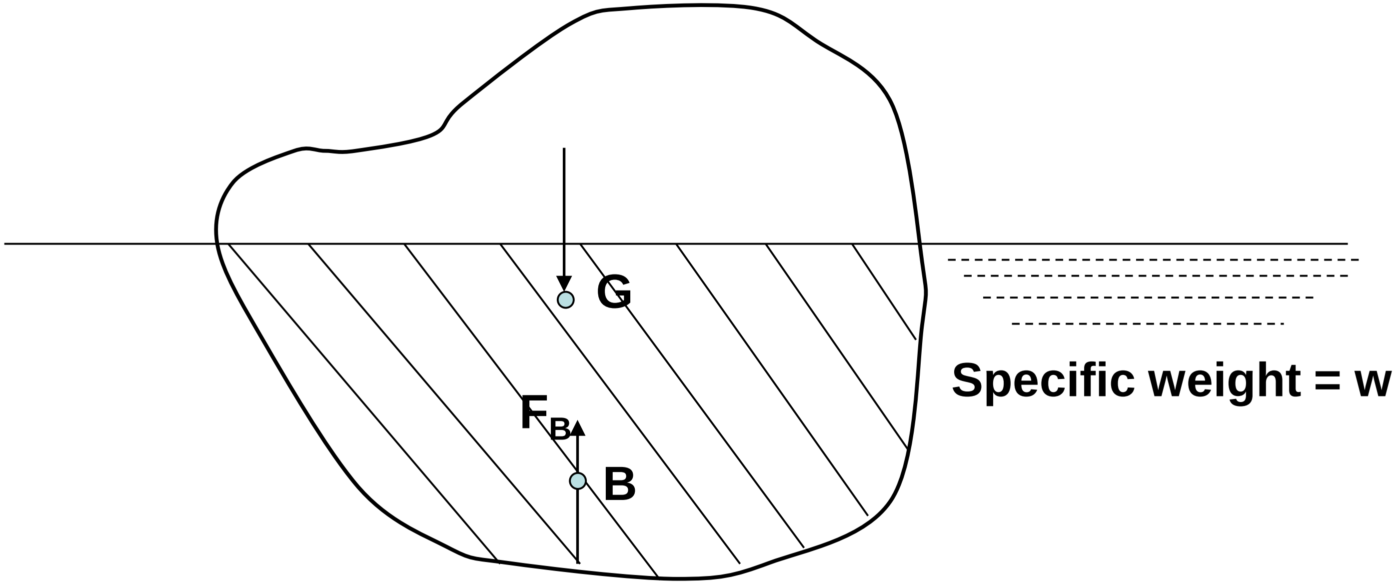
$$= (w_1 y_1 + w_2 y_2) dA$$

$$= w_1 d\overline{V_1} + w_2 d\overline{V_2}$$

$$F_B = (w_1 \overline{V_1} + w_2 \overline{V_2})$$

Buoyant Force on Body Floating in Air

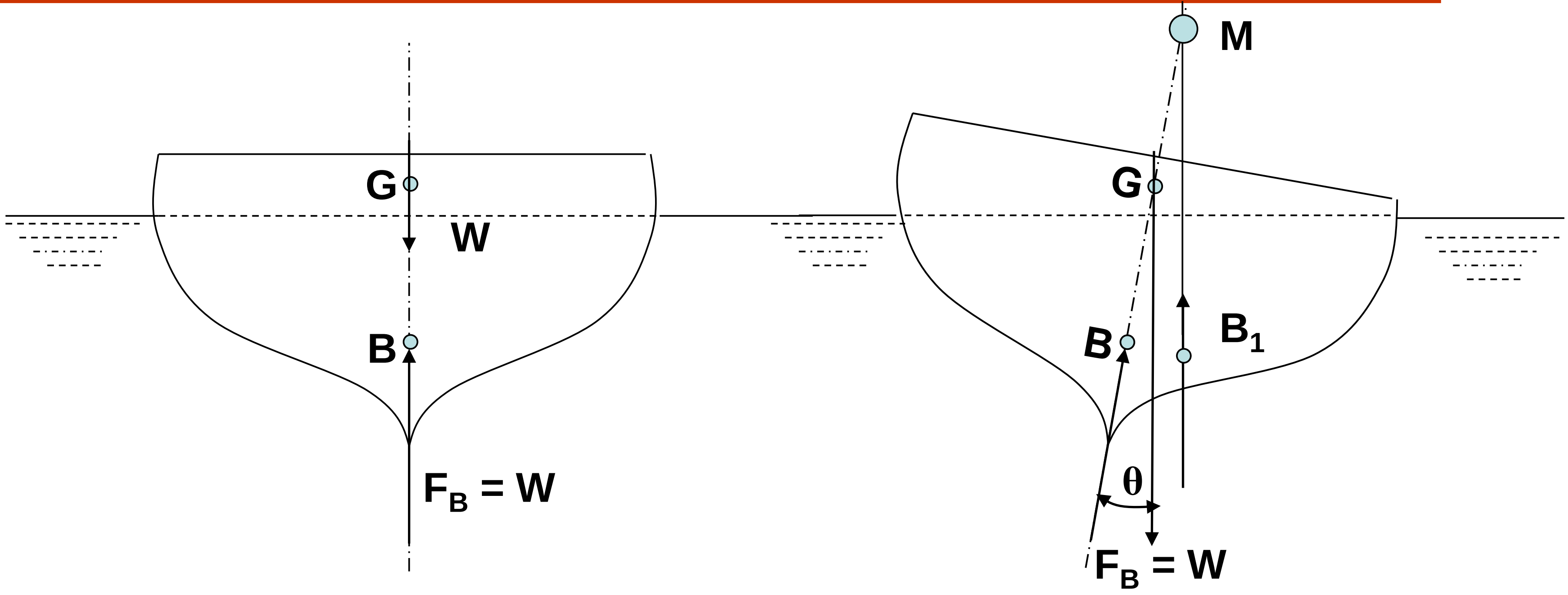
The weight of the air displaced by the body can be neglected as specific weight of air is negligible



$$F_B = w\overline{V} = W$$

W is the weight of liquid displaced

Metacentre and Metacentric Height



- Metacentre is defined as the point of intersection between the axis of the floating body passing through points B and G and a vertical line passing through the new centre of buoyancy B_1.
- For small θ the position of M is practically same.

- The distance between the centre of gravity G and the metacentre M of a floating body (i. e. GM) as $\theta \rightarrow 0$, is known as metacentric height

Stability of Submerged and Floating Bodies

Stability of submerged or floating body

- Tendency of the body to return to the original upright position after it has been slightly displaced.

When a submerged or floating body is given a slight angular displacement it may have either of the following three conditions of equilibrium

- **Stable equilibrium**
- **Unstable equilibrium**
- **Neutral equilibrium**

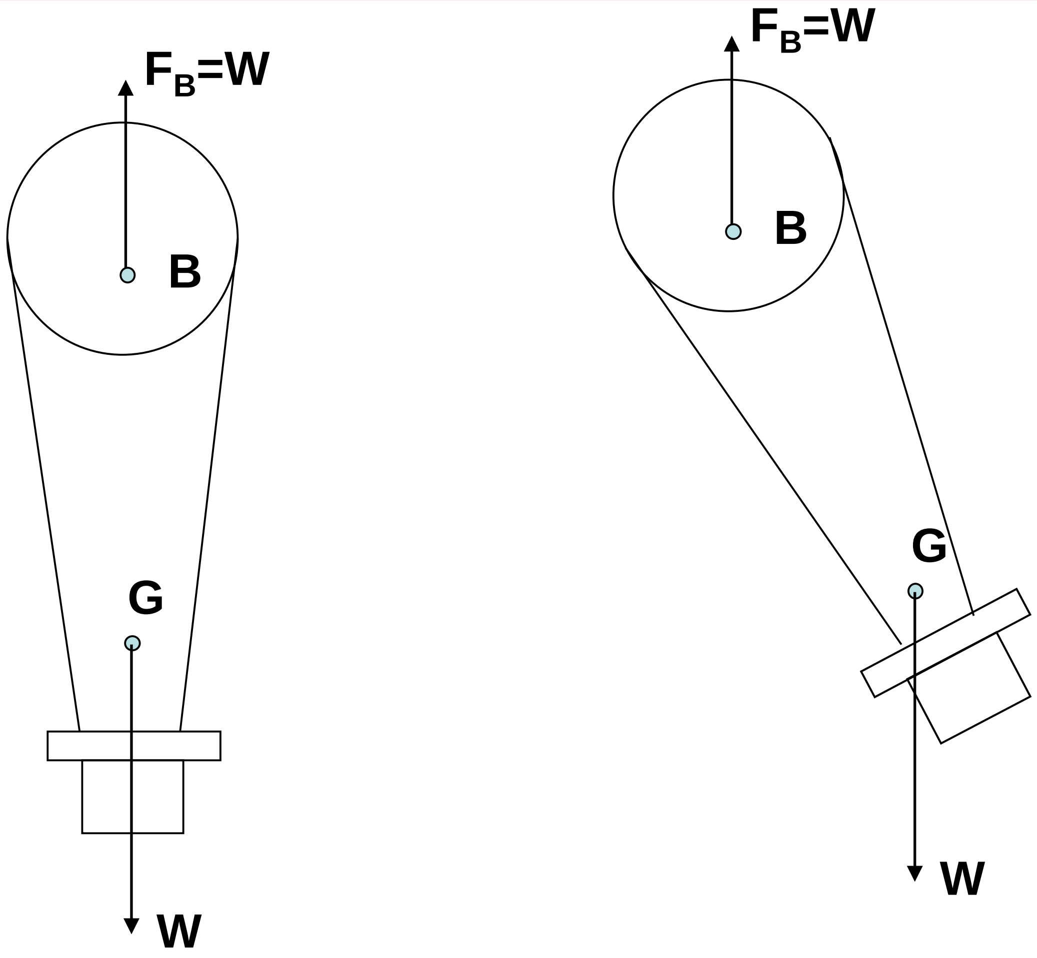
Stability of Submerged and Floating Bodies

A body is said to be in a state of stable equilibrium if small angular displacement sets up a couple which oppose the angular displacement and brings back the body to its original position.

A body is said to be in a state of unstable equilibrium if small angular displacement sets up a couple which tends to further increase the angular displacement and thereby not allowing the body to its original position.

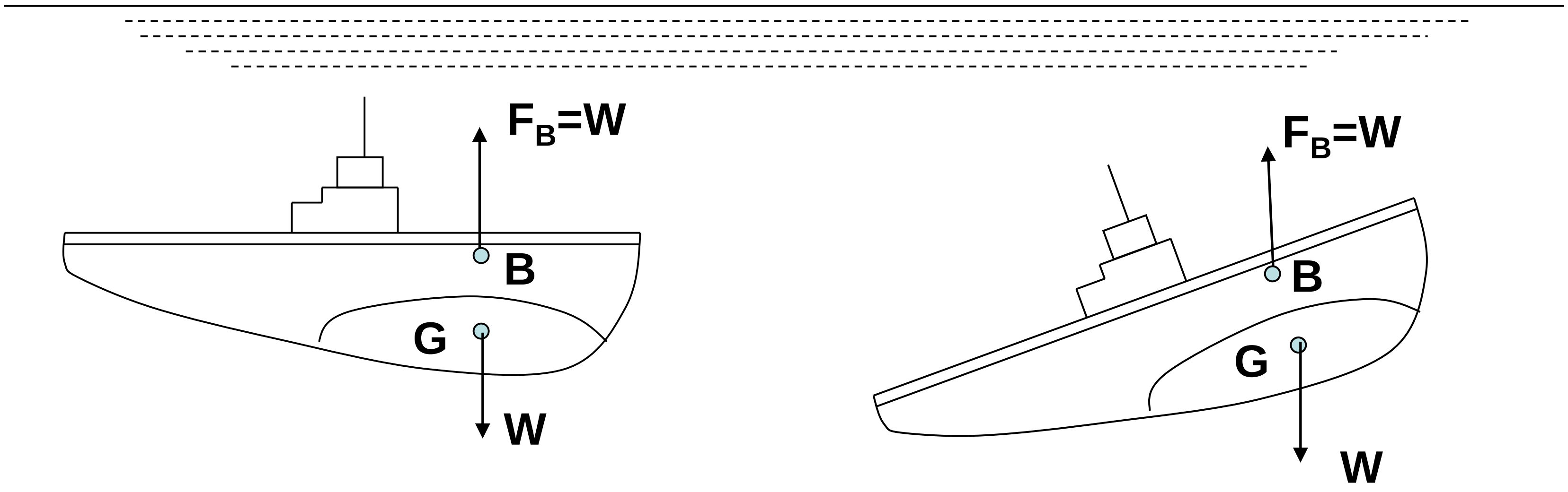
A body is said to be in a state of neutral equilibrium if small angular displacement does not set up a couple of any kind and therefore the body adopts a new position given to it by angular displacement.

Stability of Fully Submerged Body



Balloon floating in a air

Stability of Fully Submerged Body

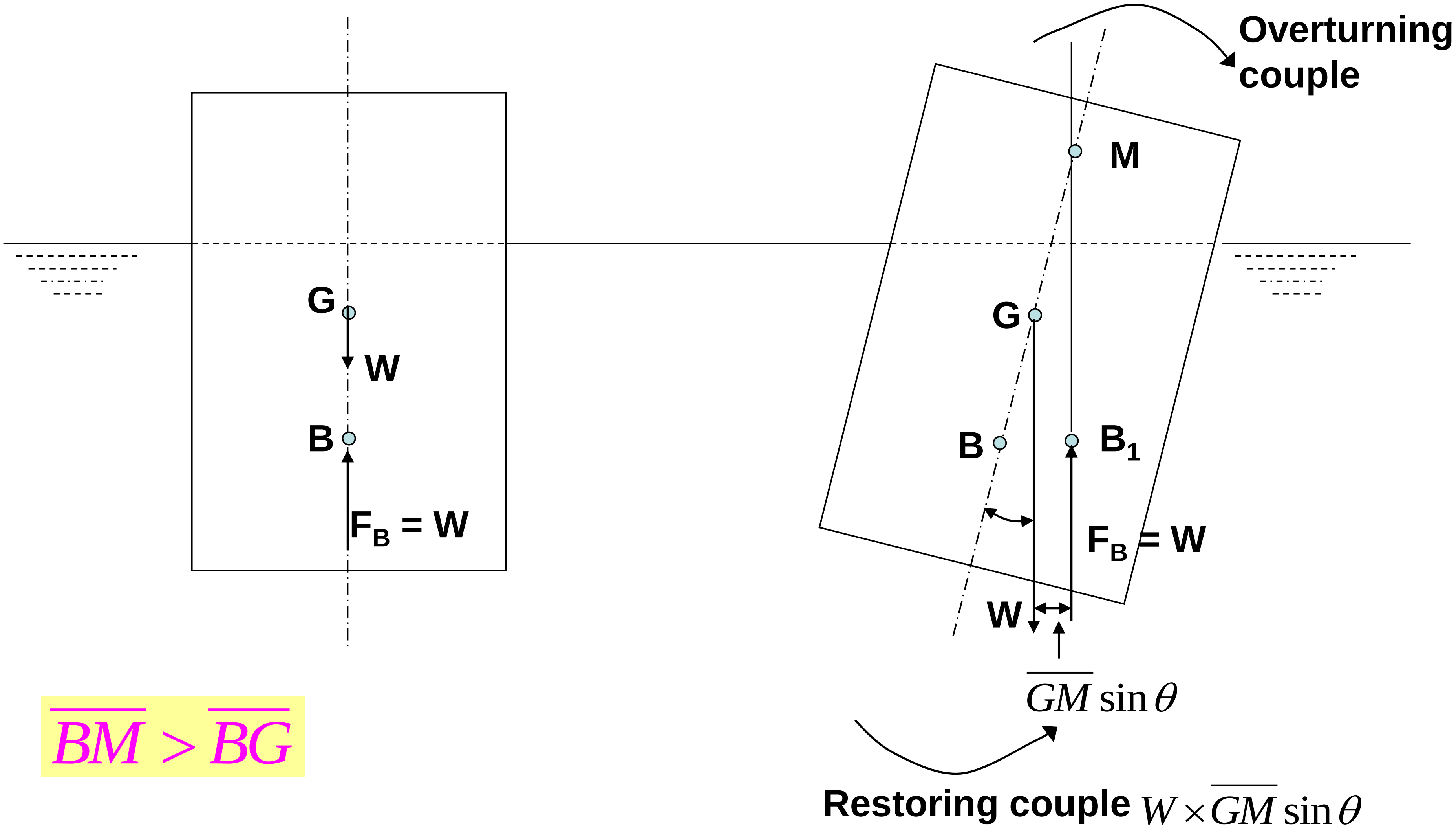


Submarine floating in sea

Condition for stable equilibrium

The fully submerged body is in stable equilibrium if centre of gravity is below centre of buoyancy

Stability of Partially Submerged Body



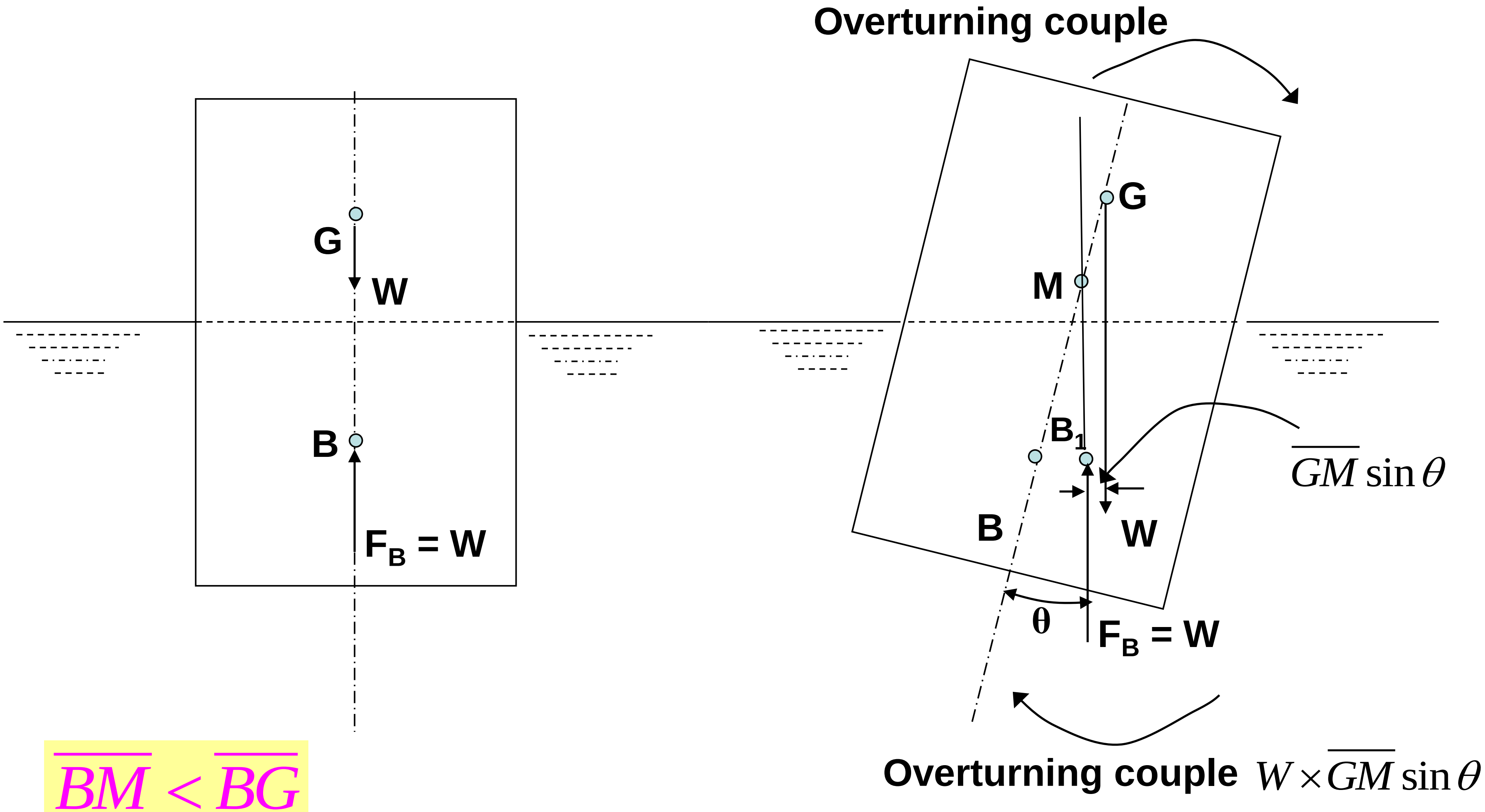
Overturning couple

Restoring couple $W \times \overline{GM}\sin\theta$

$$\overline{BM} > \overline{BG}$$

Floating body in a stable equilibrium

Stability of Partially Submerged Body



$$\overline{BM} < \overline{BG}$$

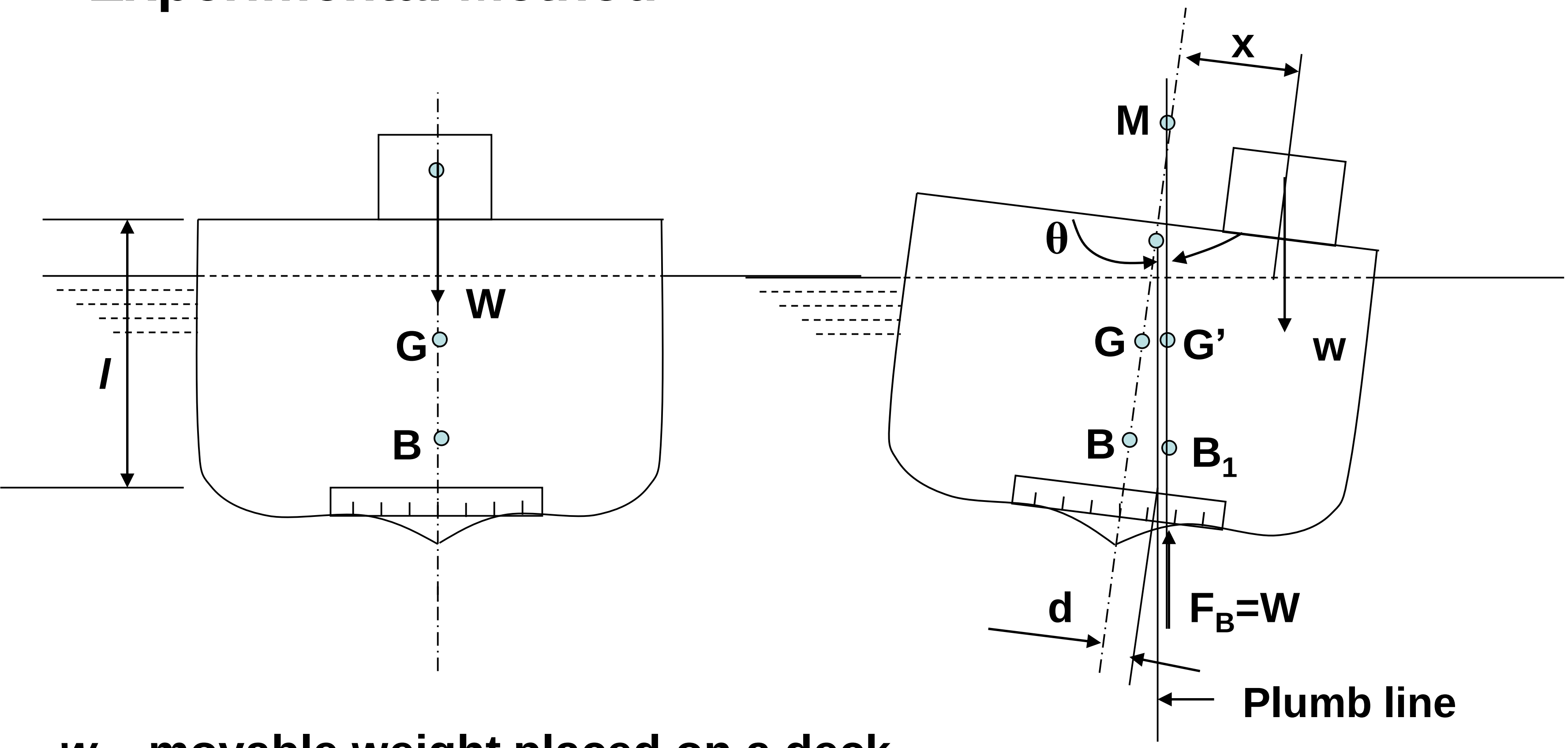
Floating body in a unstable equilibrium

Importance of Stability of Floating Objects

- Boats, ships etc are the important objects which are subjected to external forces
- Wind forces, wave forces, pressure due to tidal or river currents, pressure due to maneuvering a boat or ship in a curved path
- Shifting of cargo may cause heeling
- Movements of passengers also develops overturning couple
- Thus the care has to be taken in the design of boats or ship so that metacentre is kept well above centre of gravity
- CG can be lowered by permanently loading the ship or boat

Determination of Metacentric Height

Experimental Method



w – movable weight placed on a deck

W – Total weight including weight of deck

Determination of Metacentric Height

$$wx = W(\overline{GG^{'}})$$

$$(\overline{GG^{'}}) = (\overline{GM})\tan\theta \quad \Longrightarrow \quad W(\overline{GM})\tan\theta = wx$$

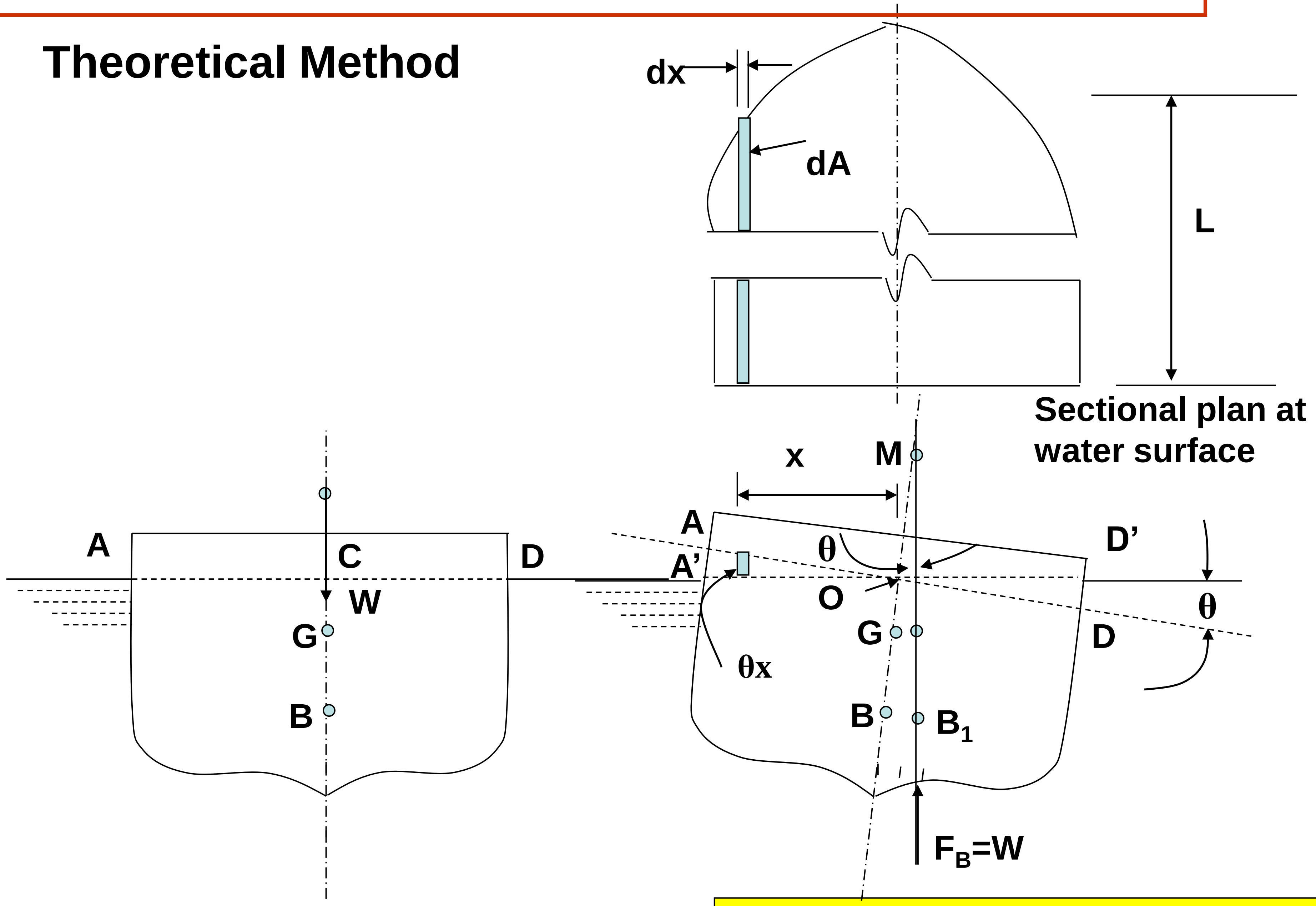
Thus $$(\overline{GM}) = \frac{wx}{W\tan\theta}$$

$$\tan\theta = \frac{d}{l}$$

$$(\overline{GM}) = \frac{wx}{W} \times \frac{l}{d}$$

Determination of Metacentric Height

Theoretical Method



Determination of Metacentric Height

- In the tilted position the portion AOA’ has emerged out
- The portion DOD’ has moved down in the liquid
- Assume that there is no vertical movement
- Volume corresponding to AOA’ and DOD’ is equal

The moment of buoyant force is $F_B \times \overline{BM} \times \theta$

Volume of each prism $= L\theta x dx$

Weight of the liquid in each prism $= wL\theta x dx$

Moment of pair of the forces due to emrging and going down of wedges $= 2x \times wL\theta x dx$

Determination of Metacentric Height

$$F_B \times \overline{BM} \times \theta = 2w\theta\int x^2 L dx = w\theta 2\int x^2 dA$$

$dA = Ldx$

$2\int x^2 dA$ ⇒ **Moment of inertia I of the cross sectional area of ship at water surface about its longitudinal axis**

Thus $$\overline{BM} = \frac{wI}{F_B} = \frac{wI}{wV} = \frac{I}{V}$$

V is volume of liquid displaced by ship

$$\overline{GM} = \overline{BM} - \overline{BG} = \frac{I}{V} - \overline{BG}$$

If metacentre M lies above CG, G

$$\overline{GM} = \overline{BG} - \overline{BM} = \overline{BG} - \frac{I}{V}$$

If metacentre M lies below CG, G

Determination of Metacentric Height

Thus,

$$\overline{GM} = \pm\left(\frac{I}{V} - \overline{BG}\right)$$

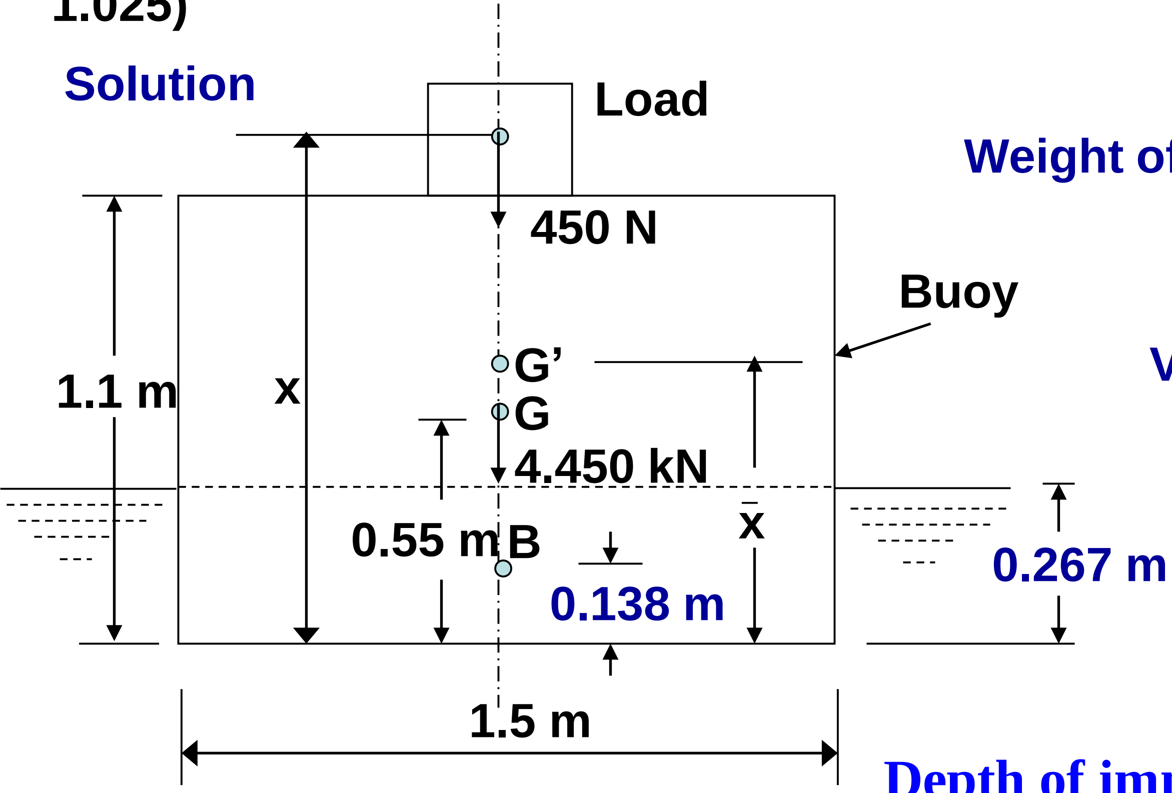
$\frac{I}{V} > \overline{BG}$ → **Stable equilibrium**

$\frac{I}{V} < \overline{BG}$ → **Unstable equilibrium**

Problem

2.1 A cylindrical buoy, diameter 1.5 m and 1.1 m high weighing 4.450 kN is floating in sea water with its axis vertical. Find the maximum permissible height above the top of the buoy, of the centre of gravity of a 450 N load which is placed centrally on top of the buoy. (Specific gravity of the seat water is 1.025)

Solution



Weight of the sea water displaced

= (4450+450) = 4900 N

Volume of sea water displaced is

$$V = \frac{4900}{1.025 \times 9810} = 0.487 \ \mathbf{m^3}$$

$$\textbf{Depth of immersion} = \frac{0487}{(\pi/4) \times 1.5^2} = 0.276 \ \mathbf{m}$$

Height of Centre of Buoyancy above the base

$$= \frac{0.276}{2} = 0.138 \ \mathbf{m}$$

Problem

The position of combined centre of gravity of buoy and the load may be obtained by taking moments total weight (acting at G') about the base of the buoy and equating it with the sum of moments of weight of buoy and weight of load about base of the buoy

Thus, $$4900 \times \overline{x} = 4450 \times \frac{1.1}{2} + 450 \times x$$

$$\Downarrow$$

$$\overline{x} = (0.499 + 0.092x)$$

The diagram suggests $$\overline{BG^{'}} = \overline{OG^{'}} - \overline{OB}$$
$$= \overline{x} - 0.138$$
$$= (0.499 + 0.092x) - 0.138$$
$$= (0.361 + 0.092x)$$

Problem

We know $\overline{BM} = \dfrac{I}{V}$ $\qquad I = \dfrac{\pi}{64}(1.5)^4 \ \mathbf{m}^4; \ V = 0.487 \ \mathbf{m}^3$

$$\Downarrow$$

$$\overline{BM} = \frac{\pi}{64} \times \frac{(1.5)^4}{0.487} = 0.510 \ \mathbf{m}$$

For stable equilibrium of the floating buoy

$$\overline{BM} > \overline{BG}^{'} \Rightarrow 0.510 > (0.361 + 0.092x) \Rightarrow 0.092x < 0.149$$

Thus, $x < 1.62 \ \mathbf{m}$

Thus the CG of the load must not be more than (1.62-1.1) = 0.51 m above the top of the buoy - answer