Introduction to Turbo Machinery

DEFINITION:

A turbo machine is a device in which energy transfer occurs between a flowing fluid and rotating element due to dynamic action. This results in change of pressure and momentum of the fluid.

The word turbo or turbines is of Latin origin, it means which spins or whirls around.

TYPE:

If the fluid transfers energy for the rotation of the impeller, fixed on the shaft, it is known as **power generating turbo machine.**

If the machine transfers energy in the form of angular momentum fed to the fluid from the rotating impeller, fixed on the shaft, it is known as **power absorbing turbo machine.**

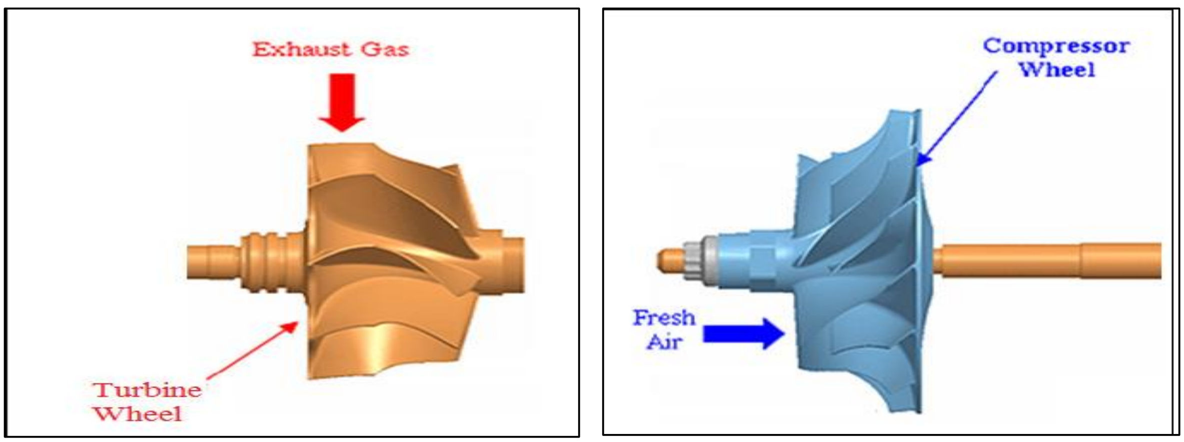


Fig. 1. Turbine and Compressor

CLASSIFICATION OF TURBO MACHINES

1. Based on energy transfer

a) Energy is given by fluid to the rotor - Power generating turbo machine E.g. Turbines

b) Energy given by the rotor to the fluid – Power absorbing turbo machine

c) E.g. Pumps, blowers and compressors

2. Based on fluid flowing in turbo machine

a) Water

b) Air

c) Steam

d) Hot gases

e) Liquids like petrol etc.

3. Based on direction of flow through the impeller or vanes or blades, with reference to the axis of shaft rotation

a) Axial flow – Axial pump, compressor or turbine

b) Mixed flow – Mixed flow pump, Francis turbine

c) Radial flow – Centrifugal pump or compressor

d) Tangential flow – Pelton water turbine

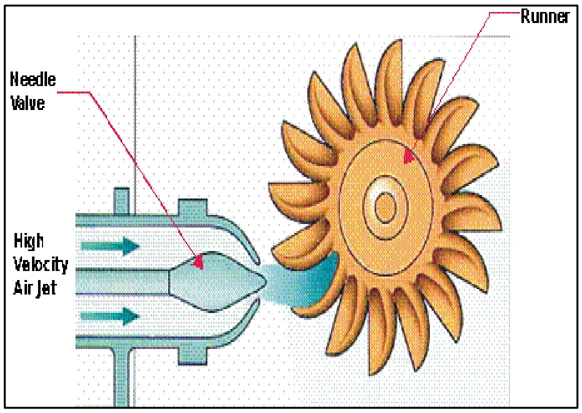


Fig. 2. Pelton Turbine

Fig. 3. Francis Turbine Runner

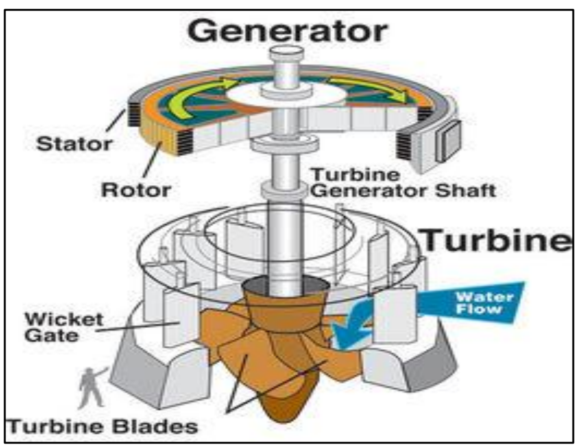


Fig. 4. Modern Francis Turbine

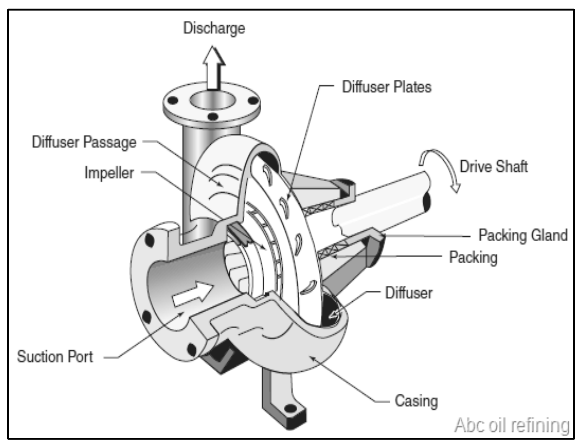


Fig. 5. Centrifugal Compressor

4. Based on condition of fluid in turbo machine

a) Impulse type (constant pressure) E.g Pelton water turbine

b) Reaction type (variable pressure) E.g. Francis reaction turbine

5. Based on position of rotating shaft

a) Horizontal shaft – Steam turbines

b) Vertical shaft – Kaplan water turbines

c) Inclined shaft – Modern bulb micro-hydel turbines

Comparison between Positive Displacement Machines and Turbo Machines

Action:

A positive displacement machine creates thermodynamic and mechanical action between near static fluid and relatively slow moving surface and involves in volume change and displacement of fluid as in IC engines.

A turbo machine creates thermodynamic and dynamic action between flowing fluid and rotating element involving energy transfer with pressure and momentum changes as shown in gas turbines.

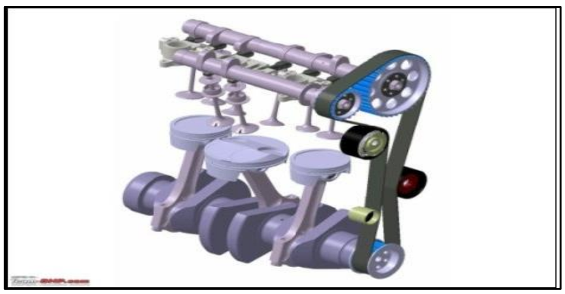


Fig. 6. Positive displacement machine

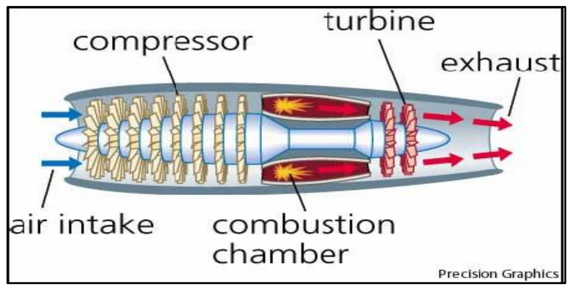


Fig. 7. Aircraft jet propulsion system

Operation:

The positive displacement machine commonly involves reciprocating motion and unsteady flow of fluids like in reciprocating IC engines or slow rotating fluids like in gear pumps.

A turbo machine involves steady flow of fluid with pure rotary motion of mechanical elements. Only unsteadiness will be there during starting, stopping and changes in loads on the machine.

Mechanical features:

A positive displacement machine commonly work at low speeds and involves complex mechanical design. It may have valves and normally will have heavy foundation.

A turbo machine works at high speeds, simpler in design, light in weight, have less vibration problems and require light foundation.

Efficiency of energy conversion:

A positive displacement machine gives higher efficiency due to energy transfer near static conditions either in compression or expansion processes.

A turbo machine gives less efficiency in energy transfer. The energy transfer due to dynamic action will be less during compression process of fluid like pumps and compressors and will be slightly more during expansion processes like in turbines but still lower than reciprocating machines.

Volumetric efficiency:

The volumetric efficiency of a positive displacement machine is low due to closing and opening of the valves during continuous operation.

In turbo machines, since there are no valves under steady flow conditions, the volumetric efficiency will be close to 100 per cent. A turbo machine has high fluid handling capacity.

Weight to mass flow rate:

A reciprocating air craft IC engine power engine developing 300 KW handles 2 kgs/sec of air weigh's around 9500 N. Where, as a rotary gas turbine of an air craft for same 300 KW power can handle 22 kgs/sec of air and weighs only 8000N handling more mass of air/sec. In stationary power plants, the specific weight of reciprocating power plants will be 10-15 times higher than the turbo power plants.

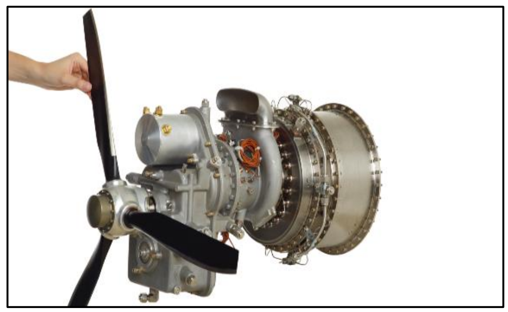
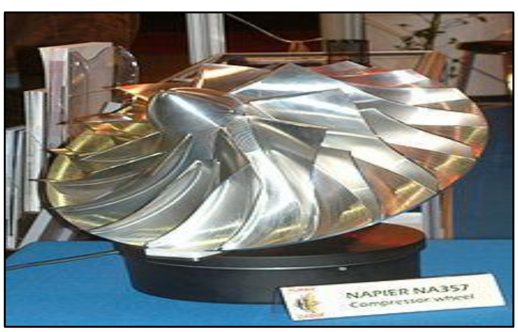


Fig. 8. A reciprocating air craft IC engine and a rotary gas turbine

Fluid phase:

Turbo machines have the phase changes occurring in fluid like cavitation in hydraulic pumps and turbines and surge and stall in compressors, blowers and fans if the machines are operated at off design condition leading to associated vibrations and stoppage of flow and damage to blades.

Positive displacement machines have no such problems



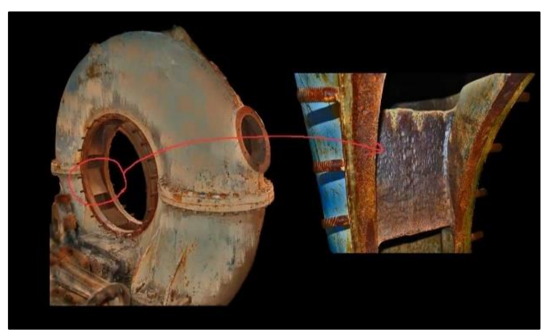


Fig. 9. Compresssor Blades and Cavitation in Francis Turbine

Application of Turbomachines

1. Electric Power generation (Pelton, Francis and Kaplan Turbines)
2. Thermal Power Plants (Steam Turbines)
3. Wind Mills
4. Aircraft Propulsion
5. Multi stage Centrifugal pumps and compressors used in Petrochemical industries
6. Fans & blowers in furnace to produce draught

Revision of Fluid mechanics

1. **Fluid :** Gas or Water
2. **Density,** $\rho = \frac{m}{V}$, kg/m^3
3. **Specific Weight:** $\gamma = \frac{W}{V} = \rho\, g$, N/m^3
4. **Pressure =** $\rho\, g\, h$, kPa or N/m^2 or bar
5. **Continuity Equation,** $\dot{m} = \rho\, Q = \rho\, AV$, kg/s

   $Q = AV$, m^3/s
6. **Euler's Equation =** $\int \frac{dp}{\rho} + V\, dv + g\, dz = 0$
7. **Work in Linear Motion, W = F d , N-m**
8. **Torque in Rotary Motion, T = F r, N-m**
9. **Rate of doing work is called Power**

   Linear motion, Power = $\frac{F\, d}{t} = F\, v$, Watt

   Rotary motion, Power = T ω = $\frac{2\pi\, N\, T}{60}$ in Watt
10. Turbine Power, $P_t = F\, u = \frac{\dot{m}\,(V1 - V2).u}{1000}$ in KW
11. Compressor Power, $P_c = F\, u = \frac{\dot{m}\,(V2 - V1).u}{1000}$ in KW
12. **Efficiency =** $\frac{Output}{Input} = \frac{Workdone/sec}{Kinetic\ Energy/sec} = \frac{Workdone/sec}{Kinetic\ Energy/sec}$

    Kinetic Energy = $\frac{1}{2}\, m\, V^2$

    Kinetic Energy/sec = $\frac{1}{2}\, \dot{m}\, V^2$

Rate of Flow or Discharge:

The quantity of fluid flowing across a section in unit time is called rate of flow.

(i) Volume Flow Rate (Q):

It is the volume of fluid flowing across the section in unit time.

Unit: m^3/s or lpm or lps

1000 litre/sec = 1 m^3/s

60000 litre/min = 1 m^3/s

Q = Area of section x Velocity of flow

Q = AV

(ii) Mass Flow Rate ($\dot{m}$) Kg/s:

It is the mass of fluid flowing across the section in unit time.

$\rho = \frac{m}{V} = \frac{m/t}{V/t} = \frac{\dot{m}}{Q}$

Unit: kg/s

(iii) Weight Flow Rate ($\dot{w}$) N/s:

It is the weight of fluid flowing across a section in unit time.

Unite: N/s

Relationship among $\dot{w}$, $\dot{m}$ and Q

Weight = $\frac{Weight}{Volume}$

Weight = Weight density x Volume

Weight of fluid flowing/second = Weight density x Volume of fluid flowing/second

$\dot{w} = \gamma \times Q$

$\dot{w} = \gamma AV$

Weight = Mass x g

Weight flowing per second = Mass flowing per second x g

$\dot{w} = \dot{m} \times g$

Mass density = $\frac{Mass}{Volume}$

Mass = Mass density x Volume

Mass of fluid flowing/second = Mass density x Volume of fluid flowing/second

$\dot{m} = \rho \times Q$

$\dot{m} = \rho AV$

Newton's II Law of Motion:

"The rate of change of momentum of a moving body is directly proportional to the magnitude of the applied force and takes place in the direction of the applied force".

F α $m.\frac{(V2-V1)}{t}$

F α m.a

F = k. m.a

If m = 1 kg, a= 1m/s^2 and F= 1N

Therefore, k = 1

Then, F = m.a

S.I Unit of Force is Newton (N)

Momentum

The capacity of a moving body to impart motion to other bodies is called momentum.

The momentum of a moving body is given by the product of mass and velocity of the moving body.

Momentum = Mass x Velocity = m x V

Unit: kg.m/s

Impulsive Force and Impulse of Force:

A force acting over a short interval of time on a body is called impulsive force.

Eg: Kick given to a football.

Impulse of a force is given by the product of magnitude of force and its time of action.

Impulse of a force = Force x Time interval

SI unit: Ns

Impulse – Momentum Principle:

From Newton's II Law

F = ma

F = $m.\frac{\boldsymbol{dV}}{\boldsymbol{dt}} = \frac{\boldsymbol{d(m.V)}}{\boldsymbol{dt}}$

F.dt = d(M)

Impulse = Change in momentum

Or

F = $m.\frac{(V2-V1)}{t}$

$Ft = mV_2 - mV_1$

Impulse = Final momentum – Initial momentum

Impulse of a force is given by the change in momentum caused by the force on the body.

Impulse momentum principle states that the impulse exerted on any body is equal to the resulting change in momentum of the body

$F = \frac{m}{t}(V_2 - V_1) = \dot{m}(V_2 - V_1)$

F = ṁ x Final velocity – m x Initial velocity

$F = \dot{m}(V_2 - V_1) = \rho Q (V_2 - V_1) = \rho A V (V_2 - V_1)$

V_2 = Final velocity of fluid along the direction.
V_1 = Initial velocity of fluid along the direction.

$F_x = \dot{m}(V_{2x} - V_{1x})$

$F_y = \dot{m}(V_{2y} - V_{1y})$

The equation represents the force exerted by the body on the fluid.

Therefore, the force exerted by the fluid on the body will be equal in magnitude and opposite in direction, by using Newton's II law of motion.

Hence, the force exerted by the fluid on the body is

$F = -[\rho Q (V_2 - V_1)]$

$F = \rho Q (V_1 - V_2)$

FLAT PLATE:

(A) Force exerted by jet on Fixed Vertical Plate:

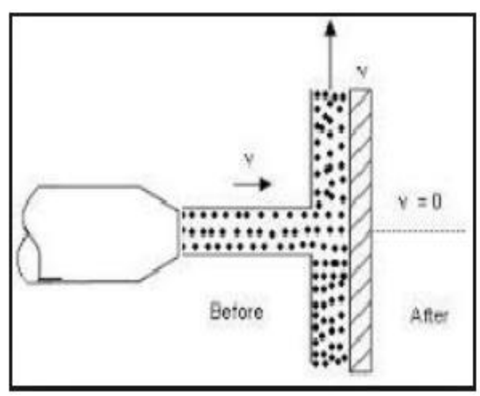


Fig. 10. Impulse Momentum Principle

Let,

V = velocity of jet, m/s

d = diameter of jet, m

a = area of jet = $\frac{\pi}{4}d^2$, m^2

u = velocity of plate, m/s

u = 0 m/s for fixed flat plate

F_x = force exerted by jet

F_x = Rate of change of momentum in the direction of flow.

$= \frac{Initial\ Momentum - Final\ Momentum}{Time}$

$= \frac{Mass\ x\ initial\ velocity - Mass\ x\ Final\ Velocity}{Time}$

$= \frac{Mass}{Time}(V_1 - V_2)$

$= \dot{m}(V_1 - V_2)$

$= \rho a V(V - 0)$

$F_x = \rho a V^2$ (Newton)

(B) Force exerted by jet on Moving Vertical Plate:

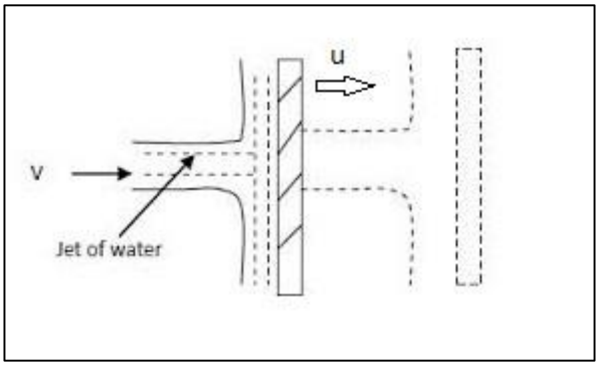


Fig. 11. Force exerted by jet on Moving Vertical Plate

When, the moving plate is vertical and moving along the Jet.

Let, V = velocity of jet (Absolute velocity)

d = diameter of jet

a = area of jet = $\frac{\pi}{4} d^2$

u = Velocity of the flat plate

Relative velocity of the jet with respect to (w.r.t.) plate = (V-u)

Mass of water striking the plate = $\rho a(V - u)$

Therefore force exerted by jet on moving plate in the direction of jet,

F_x= mass of water striking/sec x (initial velocity – Final Velocity)

$= \rho a(V - u)[(V - u) - 0]$

$F_x = \rho a(V - u)^2$

In this case there will be work done/sec by jet on the plate.

W.D/sec = F x Distance travelled by the plate in the direction of force/sec

W.D/sec = $\rho a(V - u)^2$ x u N-m/s (i.e. watts)

INCLINED PLATE

(A)Force exerted by jet on Fixed Inclined Plate:

The configuration of the system is shown in following fig.

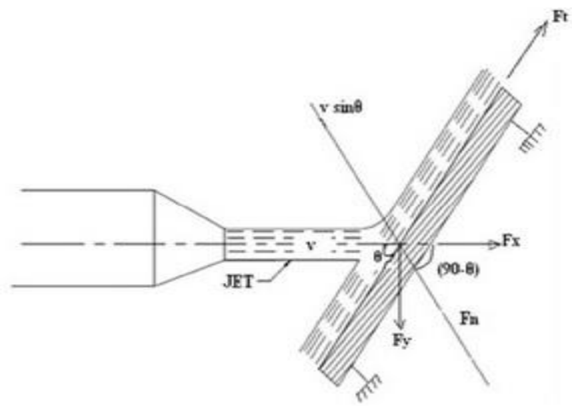


Fig. 12. Force exerted by jet on Fixed Inclined Vertical Plate

The force F_n is the force acting perpendicular to the plate when the jet with a velocity V is flowing along X direction shown in the figure can be calculated as

The component of V along N direction= $V\sin\theta$

Where $\theta = is\ the\ angle\ between\ jet\ and\ plate$

F_n = Change in momentum of the liquid along N direction.

$= \rho aV(Initial\ momentum - Final\ Momentum)$

$= \rho aV(Vsin\theta - 0)$

$F_n = \rho aV^2\sin\theta$

Force in the direction of X and Y is given as follow

$F_x = F_n\cos(90\text{-}\theta) = F_n\sin\theta$

$= \rho aV^2\sin\theta\ sin\theta$

$F_x = \rho aV^2 sin^2\theta$

$F_y = F_n\sin(90\text{-}\theta) = F_n\cos\theta$

$F_y = \rho aV^2\sin\theta\ cos\theta$

$= \frac{\rho aV^2 Sin2\theta}{2}$

$F_n = \sqrt{F_x{}^2 + F_y{}^2}$

Work Done= F_x x u = 0 (Since plate is stationary, u = 0)

Discharge: Ratio of discharge

As the jet strikes the plate, it gets divided into two portions Q_1 and Q_2 as shown in fig.

If no loss is considered,

Initial Momentum along the axis of the plate = Momentum of two streams along the axis of the plate.

$$Q\rho V cos\theta = Q_1\rho V - Q_2\rho V$$

Q_1-Q_2 = Q$\cos\theta$ ------------------------------------(1)

Q_1 + Q_2 = Q ------------------------------------(2)

Adding equations 1 and 2 we get

$2Q_1 = Q\cos\theta + Q$

$Q_1 = \frac{Q}{2}(1+ \cos\theta)$

Similarly,

$Q_2 = \frac{Q}{2}(1- \cos\theta)$

Therefore,

$$\frac{Q_1}{Q_2} = \frac{1+cos\theta}{1-cos\theta}$$

(B)Force exerted on inclined plate moving in the direction of Jet

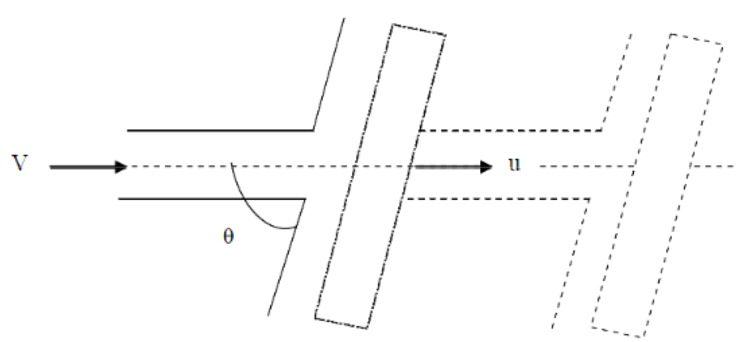


Fig. 13. Force exerted by jet on inclined plate moving in the direction of Jet

Let,

V= Absolute velocity of jet

u = Velocity of plate

θ = Angle between jet and plate

Relative velocity of jet = (V-u)

Mass of water striking/sec = $\rho a(V-u)$

If there is no loss of energy the jet of water will leave the inclined plate with velocity (V-u)

Force exerted by jet in the direction normal to the plate

F_n = Mass/sec $[Initial\ velocity\ in\ normal\ direction - Final\ velocity]$

$= \rho a(V-u)[(V-u)sin\theta - 0]$

$$F_n = \rho a(V-u)^2 sin\theta$$

F_n can be resolved in two components F_x and F_y

$F_x = F_n \cos(90 - \theta) = F_n \sin\theta$

$$F_x = \rho a(V-u)^2 sin^2\theta$$

$F_y = F_n \sin(90 - \theta) = F_n \cos\theta$

$$F_y = \rho a(V-u)^2 sin\theta cos\,\theta$$

Work done per second in the direction of jet = F_x x Distance/second in the direction X

W.D/sec = F_x x u

$$W.D/sec = \rho a(V-u)^2 sin^2\theta\ x\ u \text{ N-m/s (Watts)}$$

Impact of Jet on series of flat plates Mounted on a wheel

The configuration of the system is shown in the fig. In this case the plate fixed on the wheel moves with a velocity u in the direction of jet.

As one plate comes out of the jet, the jet strikes on the other plate and whole flow from the jet strikes the plate when all plates are considered.

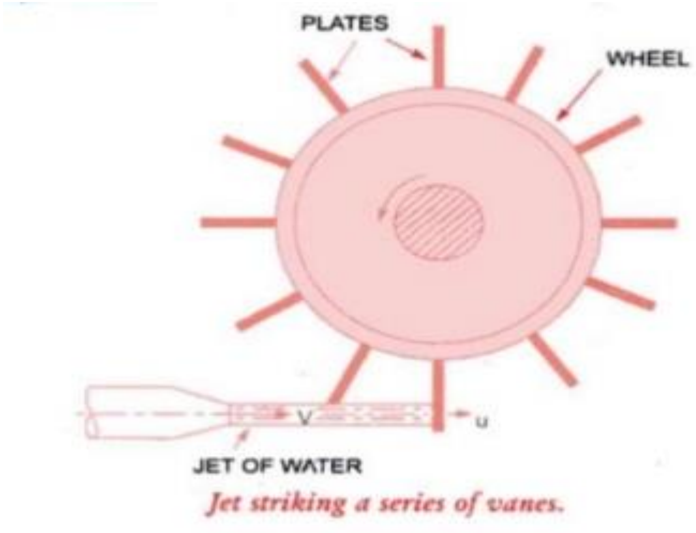


Fig. 14. Jet striking series of vanes

The velocity of the jet with which jet strikes the plate is the relative velocity of the jet which is (V-u)

The final velocity of jet after striking the plate = V-u =0.

Change in velocity = initial relative velocity – Final relative velocity

= (V-u) – 0= V-u

Force acting on the plate = $\dot{m}\ x\ Change\ in\ velocity$

$= \rho a V(V-u)$ Newtons

The efficiency of the system is given by

$$\eta = \frac{Output}{Input} = \frac{F\ x\ u}{\frac{1}{2}mV^2} = \frac{\rho a V(V-u)u}{\frac{1}{2}\rho a V V^2}$$

$$\eta = \frac{2(V-u)u}{V^2}$$

For finding out the maximum velocity for the given input (V is constant),

The condition required is, $\frac{d\eta}{du} = 0$

$$\frac{d}{du}[u(V-u)] = 0$$

V- 2u = 0

$$u = \frac{V}{2}$$

Corresponding maximum efficiency is given by

$$\eta_{max} = \frac{2\,(2u-u)x\,u}{(2V)^2}$$

$$= 0.5$$

$$\boxed{\eta_{max} = 50\%}$$

Curved Plate

1) **Force exerted by jet on stationary curved plate**

Case 1: Jet strikes the curved plate at the center of plate

Let, θ = angle made by tangent drawn at tip with direction of jet

$(180 - \theta)$ = Angle of deflection of jet with direction of jet

V = Velocity of jet at outlet

As plate is stationary inlet velocity is zero.

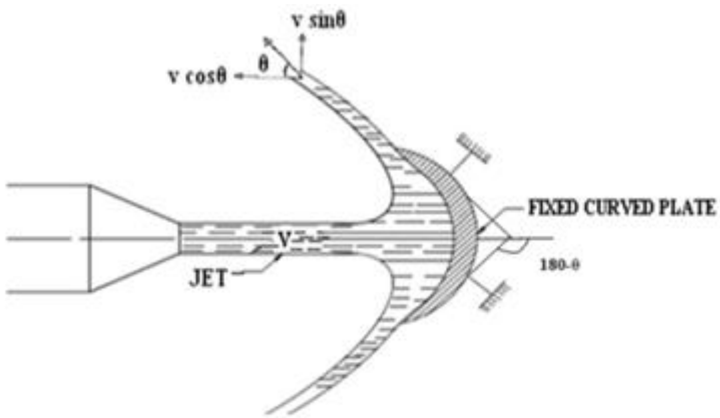


Fig. 15. Jet striking the curved plate at the center of plate

The velocity coming out of jet can be resolved in two components

1. Components of velocity in the direction of jet = - V $\cos\theta$
2. Components of velocity perpendicular the direction of jet = V $\sin\theta$

Force exerted by jet (In all direction of jet)

$F_x = \rho aV$ (initial velocity – final velocity)

$= \rho aV (V_{1x} - V_{2x})$

$= \rho aV [V - (- V\cos\theta)]$

$= \rho aV^2 (1 + \cos\theta)$

$F_y = \rho aV$ (initial velocity – final velocity)

$= \rho aV (V_{1y} - V_{2y})$

$= \rho aV [0 - V \sin\theta]$

$= - \rho aV^2 \sin\theta$

(-ve sign indicates force acting in downward direction)

Resultant force $F = \sqrt{Fx^2 + Fy^2}$

$= \sqrt{\rho aV2\,(1\ +\ \cos\theta)^2 + -\rho aV2\,\sin\theta^2}$

$\varphi = \tan \frac{Fy}{Fx}$

$= \tan \frac{-sin\theta}{(1 + \cos\theta)}$

Workdone by jet per sec = F_x x u

= 0 (as initial velocity u = 0)

$\eta = \frac{workdone\ per\ sec}{kinetic\ energy/sec} = 0$

Case 2: Jet strikes the curved plate tangentially and plate is symmetrical

Let, V = Velocity of jet

θ = angle made by x-axis at inlet tip

Since plate is smooth and there are no losses, the velocity at outlet will be v

Force exerted by jet in X and Y direction

$F_x = \rho aV$ (initial velocity – final velocity)

$= \rho aV (V_{1x} - V_{2x})$

$= \rho aV [(V \cos\theta) - (- V \cos\theta)]$

$F_x = 2 \rho aV^2 \cos\theta$

$F_y = \rho av$ (initial velocity – final velocity)

$= \rho aV (V_{1y} - V_{2y})$

$= \rho aV [(V \sin\theta) - V \sin\theta)]$

$= 0$

Resultant force $F = \sqrt{Fx^2 + Fy^2}$

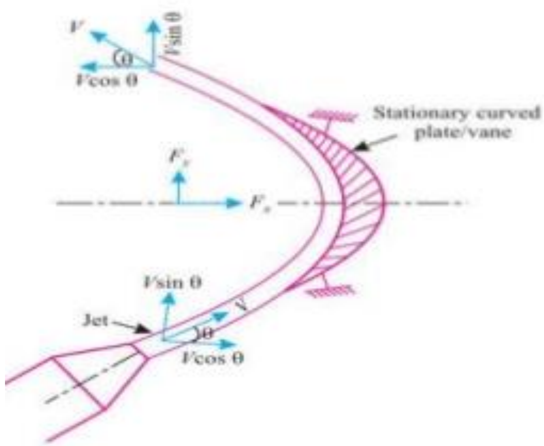


Fig. 16. Jet strikes the curved plate tangentially and plate is symmetrical

$$\varphi = \tan \frac{Fy}{Fx} = 0$$

Workdone by jet per sec = F_x x u

= 0 (as initial velocity u = 0)

$$\eta = \frac{workdone\ per\ sec}{kinetic\ energy/sec} = 0$$

Case 3: Jet strikes the curved plate tangentially and plate is unsymmetrical

Let, θ = angle made by tangent at inlet tip with x-axis

Ø = angle made by tangent at outlet tip with x-axis

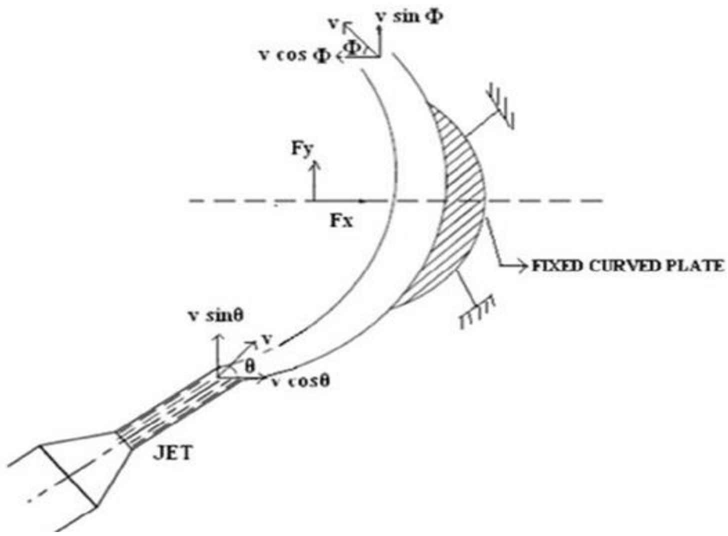


Fig. 17. Jet strikes the curved plate tangentially and plate is unsymmetrical

$V_{1x} = V \cos \theta$ $\quad\quad V_{1y} = V \sin \theta$

$V_{2x} = V \cos Ø$ $\quad\quad V_{2y} = V \sin Ø$

$F_x = \rho aV (V_{1x} – V_{2x})$

$= \rho aV (V \cos \theta - (- V \cos Ø))$

$= \rho aV^2 (\cos \theta + \cos Ø)$

$F_y = \rho aV (V_{1y} – V_{2y})$

$= \rho aV (V \sin \theta - V \sin Ø)$

$= \rho aV^2 (\sin \theta - \sin Ø)$

2) **Force exerted by jet on moving curved vane**

Case 1: Jet striking symmetrical moving curved vane at the center:

Let the curved vane of section be allowed to move with velocity u in a direction of jet.

V = Absolute velocity of jet

a = area of jet

u = velocity of plate in direction of jet

(V-u) = effective velocity of jet

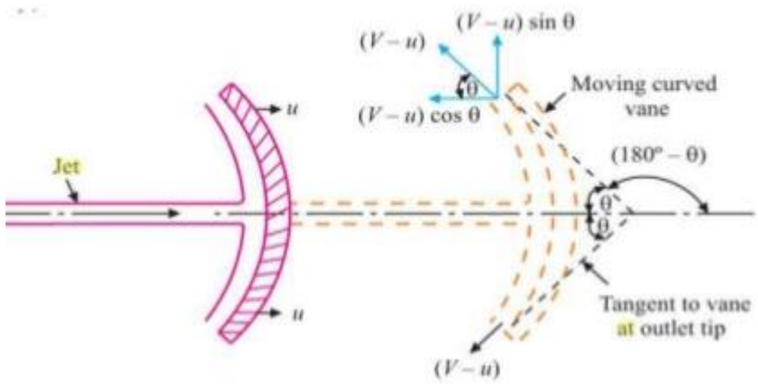


Fig. 17. Jet striking symmetrical moving curved vane at the center

Assumptions, 1) No friction between jet and plate is considered

2) Impact losses are neglected

3) No elevation difference between incoming and outgoing fluid

Component of effective velocity (V- u) is retained when fluid is leaving the curved vane tangentially.

Component of velocity leaving the jet in a direction of jet = (V-u) cos (180 – θ)

= [- (V-u) cos θ]

Mass of water striking per sec = ρa (V – u)

Force exerted by jet in the direction of jet

$F_x = \rho a$ (V-u) (initial velocity – final velocity)

$= \rho a (V-u) (V_{1x} - V_{2x})$

$= \rho a (V-u) [(V-u) - (- (V-u) \cos\ )]$

$= \rho a (V-u)^2 [1 + \cos\ ]$

Workdone on vane by jet /sec = F_x x u = $\rho a (V-u)^2 u [1 + \cos\theta]$

K.E. of jet per sec = $\frac{1}{2} m V^2 = \frac{1}{2} \rho a V^3$

Efficiency of jet = $\frac{W.D.}{K.E.} = \frac{\rho a (V-u)^2 u [1 + \cos\theta]}{\frac{1}{2}\rho a V3}$

$$\eta = \frac{2\,(V-u)^2(1+\cos\theta)u}{V^3}$$

For a given velocity of jet, efficiency is max.

If $\frac{d\eta}{du} = 0$

$\frac{d}{du}\left[\frac{2\,(V-u)^2(1+\cos\theta)u}{V^3}\right] = 0$

$\frac{2}{V^3}(1 + \cos\theta)\frac{d}{du}[(V-u)^2u] = 0$

$\frac{2}{V^3}(1 + \cos\theta)\frac{d}{du}[(V^2+u^2-2uV)\ u] = 0$

$\frac{2}{V^3}(1 + \cos\theta)\frac{d}{du}[V^2u+u^3-2u^2V] = 0$

$\frac{2}{V^3}(1 + \cos\theta)[V^2+3u^2-4uV] = 0$

$\frac{2}{V^3}(1 + \cos\theta)$ is not equal to zero,

$V^2+3u^2-4uV = 0$

$(V-u)\ (V-3u) = 0$

$u = V$ or $u = V/3$

When $u = V$, W.D. = 0

When $u = V/3$ $\eta_{max} = \frac{2\,(V-(V/3))^2(1+\cos\theta)u}{V^3}$

$= \frac{8}{27}(1 + \cos\theta)$

Thus for maximum efficiency achieved when vane velocity is 1/3 of the jet velocity.

Case 2: Jet striking unsymmetrical moving curved vane tangentially at one tip:

Let,

V_1, V_2 = Absolute velocity of jet at inlet and outlet

u_1, u_2 = peripheral velocity of vane at inlet and outlet

α, β = Angle of absolute absolute velocity of jet with direction of motion at inlet and outlet

V_{r1}, V_{r2} = Relative velocity of jet w.r.t. (V-u) at inlet and outlet

θ, Ø = Vane angle (angle made by tangent drawn to vane with direction of motion of vane) at inlet and outlet

V_{w1}, V_{w2} = velocity of whirl at inlet and outlet i.e. component of absolute velocity

V_{f1}, V_{f2} = velocity of flow at inlet and outlet

a = cross section area of jet

d = diameter of jet

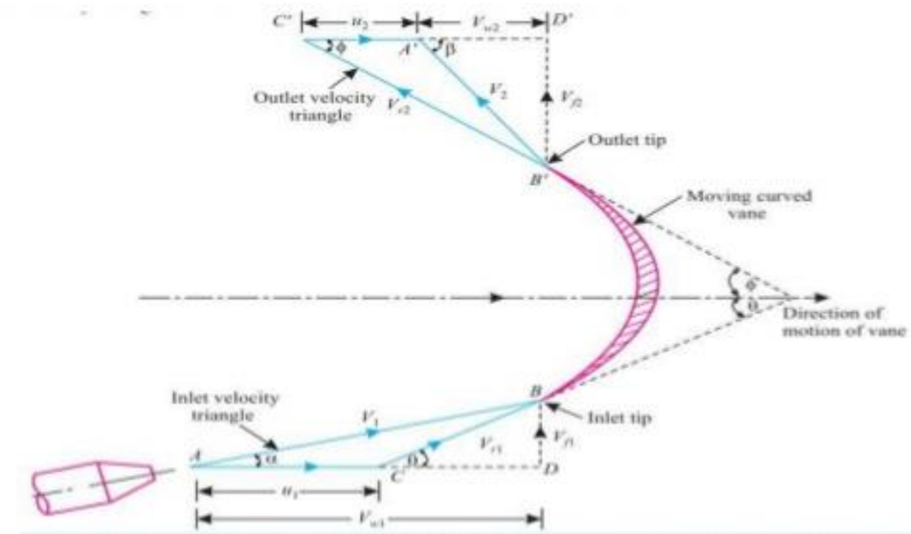


Fig. 18. Jet striking unsymmetrical moving curved vane tangentially at one tip

Assume no loss of energy during impact of jet impinges on vane tangentially and no friction loss of flow passing along smooth surface of vane

Force exerted by jet on vane in direction of motion of vane

F_x = mass of water striking/sec (horizontal component of V_{r1} in direction of motion at inlet - horizontal component of V_{r2} in direction of motion at outlet)

$= \rho a V_{r1} [V_{r1} \cos\theta - (- V_{r2} \cos Ø)]$

$= \rho a V_{r1} [V_{r1} \cos\theta + V_{r2} \cos Ø]$

$= \rho a V_{r1} [V_{w1} - u_1 + V_{w2} + u_2]$

$= \rho a V_{r1} [V_{w1} + V_{w2}]$ For $\beta < 90^0$

$= \rho a V_{r1} V_{w1}$ For $\beta = 90^0$

$= \rho a V_{r1} [V_{w1} - V_{w2}]$ For $\beta > 90^0$

W.D./sec in direction of jet = $\rho a V_{r1} [V_{w1} \pm V_{w2}]\, u$

W.D./sec per unit weight of fluid striking = $\frac{\rho a Vr1\ [Vw1 \pm Vw2]\ u}{\rho a Vr1\ g}$

$= \frac{1}{g} [V_{w1} \pm V_{w2}]\, u$ N- m/N

Efficiency of jet :

K. E. of jet = $\frac{1}{2}$ m $V^2 = \frac{1}{2}\ \rho a V^3$

$$\eta = \frac{W.D./sec}{K.E./sec} = \frac{\rho a Vr1\ [Vw1 \pm Vw2]\ u}{\frac{1}{2}\ \rho a V3}$$

Force exerted on series of radial curved vane:

Consider a series of radial curved vanes mounted on a wheel as shown in fig. The jet water impinges on the vane and the wheel starts rotating at speed

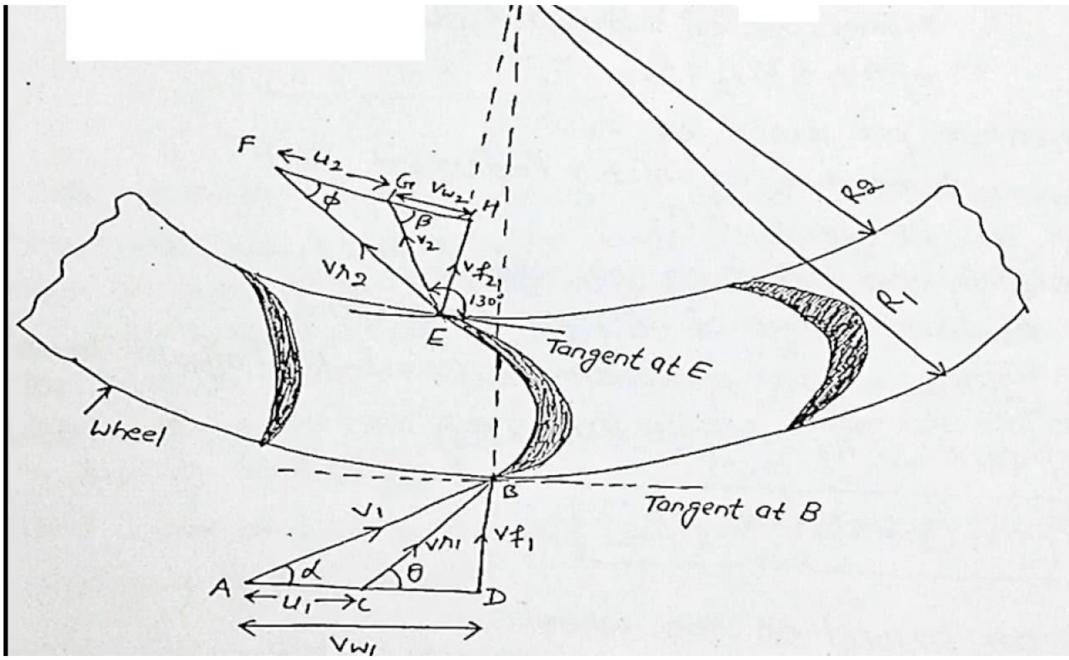


Fig. 19. Force exerted on series of radial curved vane

Let, ω = angular speed of the wheel

R_1, R_2 = Radii of the wheel at inlet and outlet of vane respectively

As vane are situated radially round the wheel blade velocities at inlet and outlet tip of vane would be different

$u_1 = \omega\, R_1$ and $u_2 = \omega\, R_2$

The flow system is inward or outward depending upon whether the jet enters the outer pheriphery or the inner periphery.

The mass of water striking/sec

= mass of water issuing from the nozzle per second

$= \rho a V_1$

Momentum of water striking the vanes (in tangential direction) per second at inlet

$= \rho a V_1 \times V_{w1}$

Similarly Momentum of water striking the vanes (in tangential direction) per second at outlet

$= \rho a V_1 \times (- V_{w2}) = - \rho a V_1 \times V_{w2}$

-ve sign is taken as V_2 at outlet is in opposite direction.

Now, angular momentum per second at inlet = momentum at inlet x radius at inlet

$= \rho a V_1 \times V_{w1} \times R_1$

angular momentum per second at outlet $= \rho a V_1 \times V_{w2} \times R_2$

Torque exerted by water on the wheel,

T = rate of change of angular momentum

= (initial angular momentum/sec - final angular momentum/sec)

$= \rho a V_1 \times V_{w1} \times R_1 - (-\rho a V_1 \times V_{w2} \times R_2)$

$= \rho a V_1 (V_{w1} \times R_1 + V_{w2} \times R_2)$

W.D./sec on the wheel = $T \times \omega$

$= \rho a V_1 (V_{w1} \times R_1 + V_{w2} \times R_2) \times \omega$

$= \rho a V_1 (V_{w1} \times \omega R_1 + V_{w2} \times \omega R_2)$

$= \rho a V_1 (V_{w1} \times u_1 + V_{w2} \times u_2)$

In case β is an obtuse angle,

then W.D./sec $= \rho a V_1 (V_{w1} \times u_1 - V_{w2} \times u_2)$

If discharge is radial at outlet, then $\beta = 90^0$

W.D./sec $= \rho a V_1 (V_{w1} \times u_1)$ $(V_{w2} = 0)$

Efficiency of radial curved vane

$$\eta = \frac{W.D./\,sec}{K.E./sec}$$

$$= \frac{\rho aV1\ (Vw1 \text{ x } u1 \pm Vw2 \text{ x } u2)}{\frac{1}{2}\rho aV3}$$

$$= \frac{2\ (Vw1 \text{ x } u1 \pm Vw2 \text{ x } u2)}{V^2}$$

Hydraulic Turbines

Hydraulic turbines are machines which use the energy of water and convert it into mechanical energy. This mechanical energy developed by a turbine is used in running electrical generator. From this electrical generator we get electric power. Hydropower is a conventional renewable source of energy which is clean, free from pollution.

Classification

According to type of energy at inlet

a) Impulse turbine: All the available energy of water is converted into kinetic energy or velocity head.
b) Reaction turbine: At inlet, water possesses kinetic energy as well as pressure energy. As the water passes through runner, pressure energy of water is converted into kinetic energy gradually.

According to name of the originator

a) Pelton turbine: It is named after Lester Allen Pelton of California (U.S.A). It is an impulse type of turbine and is used for high head and low discharge.
b) Francis turbine: It is named after James Bichens Francis. It is a reaction type of turbine from medium high to medium low heads and medium small to medium large quantities of water.
c) Kaplan turbine: It is named after Dr. Victor Kaplan. It is a reaction type of turbine for low heads and large quantities of the flow.

According to direction of flow of water in runner

a) Tangential flow turbine (Pelton turbine)
b) Radial flow turbine (no more used)
c) Axial flow turbine (Kaplan turbine)
d) Mixed (radial and axial) flow turbine (Francis turbine): It enters radially and comes out axially.

According to disposition of the turbine shaft

Turbine shaft may be either vertical or horizontal. In modern practice, Pelton turbines usually have horizontal shafts whereas the rest, especially the large units , have vertical shafts.

According to specific speed

It is defined as the speed of the geometrically similar turbine that would develop 1kW under 1m head. All geometrical similar turbines will have the same specific speeds when operating under the same head.

Specific, $N_s = \frac{N\sqrt{P}}{H^{5/4}}$

Where, N= the normal working speed

P= power output of the turbine

H=the net or effective head in metres.

Impulse turbines (Pelton Wheel)

It is an impulse turbine the pressure energy of water is converted into kinetic energy when passed through the nozzle and forms the high velocity jet of water. The formed water jet is used for driving the wheel.

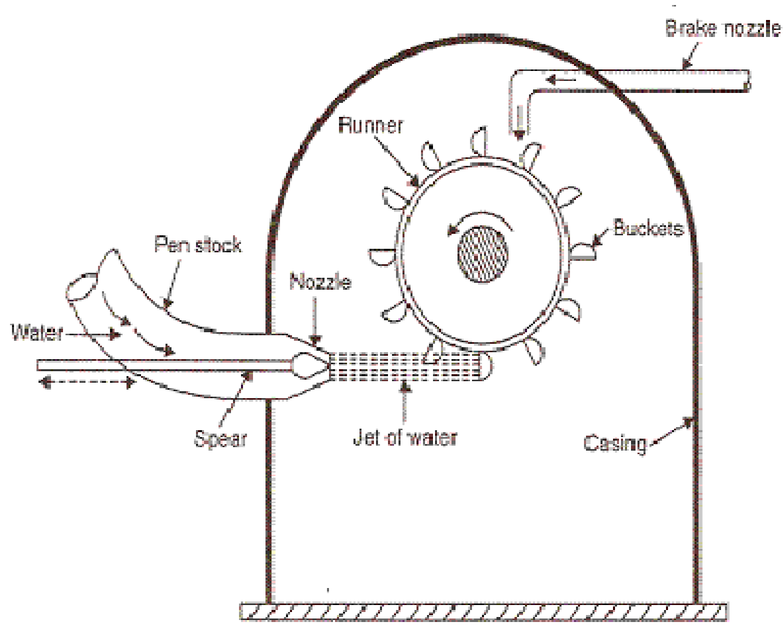
Pelton wheel among the various impulse turbine that have been designed and utilized, is by far the important. The Pelton wheel or Pelton turbine is a tangential flow impulse turbine.

Table: Important Pelton turbine Installation in India

Sr. No.	Scheme/Project	Location(State)	Source of water
1	Koyana Hydroelectric project	Koyana (Maharashtra)	Koyana river
2	Mahatma Gandhi hydroelectric works	Sharavathi (Karnataka)	Sharavathi river
3	Mandi hydroelectric scheme	Joginder Nagar (Himachal Pradesh)	Uhl river
4	Pallivasal Power station	Pallivasal (Kerala)	Mudirapuzle river
5	Pykara hydroelectric scheme	Pykara (Tamil Nadu)	Pykara river

Construction and working of Pelton turbine/wheel

A pelton wheel/turbine consists of a rotor, at the periphery of which are mounted equally spaced double hemispherical or double ellipsoidal buckets. Water is transferred from a high head source through penstock which is fitted with a nozzle, through which the water flows out as a high speed jet.



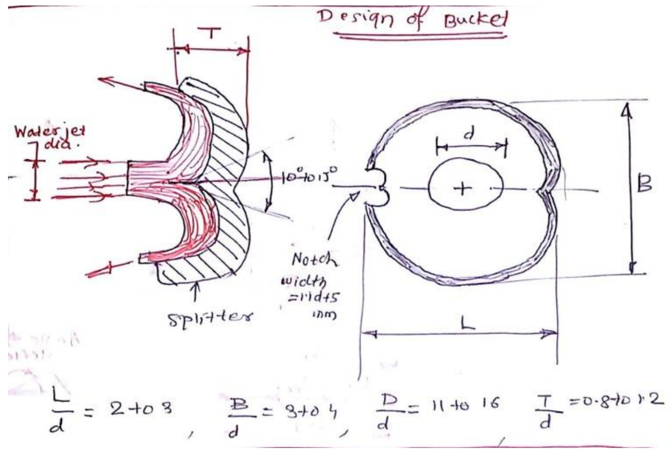
A needle spear moving inside the nozzle controls the water flow through the nozzle and at the same time, provides a smooth flow with negligible energy loss. All the available potential energy is thus converted into kinetic energy before the jet strikes the buckets of the runner. The pressure all over the wheel is constant and equal to atmosphere, so that energy transfer occurs due to purely impulse action.

The pelton turbine is provided with a casing the function of which is to prevent the splashing of water and to discharge water to the tail race.

When the nozzle is completely closed by moving the sphere in the forward direction the amount of water striking the runner is reduced to zero but the runner due to inertia continues revolving for a long time. In order to bring the runner to rest in a short time, a nozzle (brake) is provided which directs the jet of water on the back of buckets; this jet of water is called breaking jet.

Speed of the turbine runner is kept constant by a governing mechanism that automatically regulates the quantity of water flowing through the runner in accordance with any variation of load.

Dimensions of Pelton Wheel Bucket



The jet emerging from the nozzle hits the splitter symmetrically and is equally distributed into the two halves of hemispherical bucket as shown.

The bucket centre line cannot be made exactly like a mathematical cusp, partly because of manufacturing difficulties and partly because the jet striking the cusp invariably carries particles of sand and other abrasive material which tend to wear it down.

The inlet angle of the jet is therefore between 1^o and 3^o, but it is always assumed to be zero in all calculations. Then the relative velocity of the jet leaving the bucket would be opposite in direction to the relative velocity of the entering jet; This cannot be achieved in practice since the jet leaving the bucket would then strikes the back of the succeeding buckets to cause splashing and interference so that overall turbine efficiency would fall to low valves. Hence, in practice, the angular deflection of the jet in the bucket is limited to about 165^o or 170^o, and the bucket is therefore slightly smaller than a hemisphere in size.

Work done and efficiency of Pelton wheel

Let us consider,

V_1, V_2 $\rightarrow$ Absolute velocity of jet at inlet and outlet.

α, β $\rightarrow$ (Guide angles) Angle between jet directions $(V_1,\ V_2)$ and direction of motion at inlet and outlet.

$V_{r1},\ V_{r2}$ $\rightarrow$Relative velocity of jet with respect to vane at inlet and outlet.

θ, ϕ $\rightarrow$ (Vane angles) Angle made by relative velocity with direction of motion at inlet and outlet.

$V_{w1},\ V_{w2}$ $\rightarrow$Velocity of whirl at inlet and outlet.

$V_{f1},\ V_{f2}$ $\rightarrow$Velocity of flow at inlet and outlet.

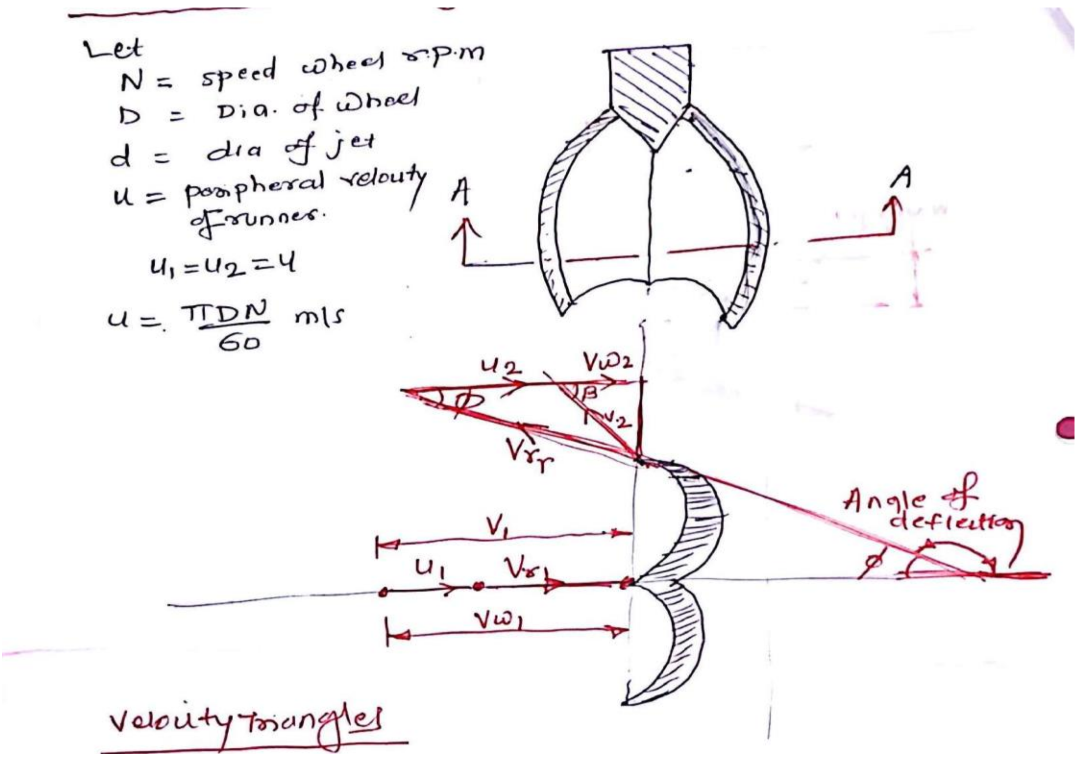


Fig. 20. Velocity triangles

Inlet velocity triangle- at inlet will be a straight line

Where $\alpha = 0, \ \theta = 0$

$u_1 = u_2 = u$ (Without shock)

$V_{r1} = V_1 - u$

$V_1 = V_{w1}......................(A)$

Outlet velocity triangle

$V_{r2} = K.V_{r1}$ i.e. $K = \dfrac{V_{r2}}{V_{r1}}$

K →blade friction coefficient, slightly less than unity.

Ideally when bucket surfaces are perfectly smooth and energy losses due to impact at splitter are neglected, $K = 1$

$V_{r2} = V_{r1}$

$Vw2 = Vr2cos\varphi - u2 \qquad for \ \beta < 90^o$

Depending upon peripheral speed u, the unit may have a slow, medium or fast runner and the angle β and Vw2 will varies as follows

1	For slow runner	$\beta < 90^o$	$Vw2 = -ve$
2	For medium runner	$\beta = 90^o$	$Vw2 = 0$
3	For fast runner	$\beta > 90^o$	$Vw2 = +ve$

The force exerted by the jet of water in the direction of motion

$$Fx = \rho a V_1 (V_{w1} + V_{w2}) \text{ ………………….. } \beta < 90^o$$

$$Fx = \rho a V_1 (V_{w1}) \text{ ……………………….. } \beta = 0^o$$

$$Fx = \rho a V_1 (V_{w1} - V_{w2}) \text{ .... } \beta > 90^o$$

a = area of jet $= \frac{\pi}{4} d^2$

Work done per sec on runner $= F_x.u$ Watt

$$= \rho a V_1 (V_{w1} + V_{w2}).u$$

Work done per sec per unit weight of water

$$= \frac{\rho a V_1 (V_{w1} + V_{w2}).u}{\rho a V_1 .g}$$

$$\frac{1}{g}(V_{w1} + V_{w2}).u \quad \text{Watt/N……………….(1)}$$

K.E of jet per sec $= \frac{1}{2}(\rho a V_1).V_1^2$

Hydraulic efficiency

$$\eta_h = \frac{WDper\sec}{K.Eper\sec}$$

$$= \frac{\rho a V_1 (V_{w1} + V_{w2}).u}{\frac{1}{2}(\rho a V_1).V_1^2}$$

$$= \frac{2((V_{w1} + V_{w2}).u}{V_1^2} \text{..........................(2)}$$

Substituting $V_{w1} = V_1$

$V_{r1} = V_1 - u$

$V_{r2} = V_1 - u$

$$V_{w2} = V_{r2}\cos\phi - u_2$$
$$= V_{r2}\cos\phi - u = (V_1 - u)\cos\phi - u$$

$$\eta_h = \frac{2(V_1 + (V_1 - u)\cos\phi - u)u}{V_1^2}$$
$$= \frac{2[(V_1 - u) + (V_1 - u)\cos\phi]u}{V_1^2}$$
$$= \frac{2(V_1 - u)(1 + \cos\phi)u}{V_1^2} \quad ....(1)$$

For maximum efficiency

$$\frac{d\eta}{du} = 0$$
$$\frac{d}{du}\left[\frac{2(V_1 - u)(1 + \cos\phi)u}{V_1^2}\right] = 0$$
$$\frac{(1 + \cos\phi)}{V_1^2}\frac{d}{du}(2(V_1 - u)u) = 0$$
$$\frac{(1 + \cos\phi)}{V_1^2} \neq 0$$
$$\therefore \frac{d}{du}(2uV_1 - 2u^2) = 0$$
$$2V_1 - 4u = 0, u = \frac{V_1}{2}$$

i.e. vane velocity is half of jet velocity.

Putting $u = \frac{V_1}{2}$

$$\therefore \eta_{\max} = 2\frac{\left(V_1 - \frac{V_1}{2}\right)(1+\cos\phi)}{V_1^2}\frac{V_1}{2}$$

$$= \frac{(1+\cos\phi)}{2}$$

For Blade friction, $K = \frac{V_{r2}}{V_{r1}}$,

For no shock, $u = u_1 = u_2$

$$\therefore V_{r2} = KV_{r1}$$

$$V_{w2} = V_{r2}\cos\phi - u2$$

Thus $= KV_{r1}\cos\phi - u$

$$= K(V_1 - u)\cos\phi - u......................(B)$$

Substituting (A) and (B) in equation (2)

$$\eta_h = \frac{2(V_1 + K(V_1 - u)\cos\phi - u).u}{V_1^2}$$

$$= \frac{2(V_1 + KV_1\cos\phi - Ku\cos\phi - u).u}{V_1^2}$$

$$= \frac{2\left[(V_1 - u)(1 + K\cos\phi)\right]u}{V_1^2}.........$$

The above equation gives Hydraulic efficiency when blade friction is considered.

Definitions of heads and efficiencies

Figure shows general layout of hydroelectric power plant using an impulse turbine.

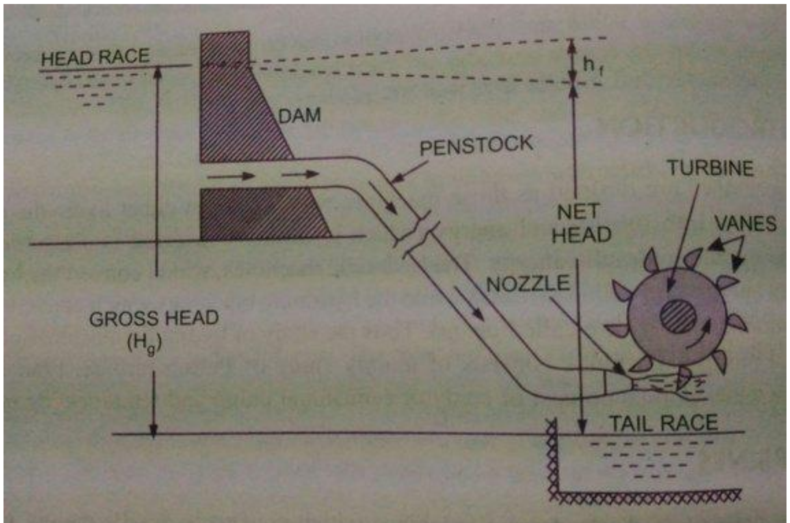


Fig. 18.1 *Layout of a hydroelectric power plant.*

1. **Gross head:** It is the difference between the water level at the reservoir and the water level at the tail race. It is denoted by H_g.
2. **Net or effective head:** The head available at the inlet of turbine is known as net or effective head. It is denoted b y $H.$ and is given by

   $H = H_g - h_f - h$

   Where $h_f =$ loss of head between head race and entrance of the turbine.

   $$h_f = \frac{4fLV^2}{2gD}$$ Where $L, V, D \rightarrow$ Penstock length, velocity and diameter

   $h =$ height of nozzle tailrace.
3. **Hydraulic efficiency:** It is defined as the ratio of power developed by runner to the power supplied by the jet to runner.

$$\eta_h = \frac{R.P}{W.P}$$

$$= \frac{W.D/\sec}{\rho.g.Q.H}$$

4. **Mechanical efficiency:** It is defined as the ratio of power obtained at shaft of turbine to the power developed by runner.

$$\eta_m = \frac{S.P}{R.P} = \frac{P}{\rho g Q H}$$

5. **Overall efficiency:** It is defined as the ratio of power available at turbine shaft to the power supplied by the water jet.

$$\eta_o = \frac{S.P}{W.P}$$
$$= \frac{S.P}{R.P} \cdot \frac{R.P}{W.P}$$
$$= \eta_m . \eta_h$$

6. **Volumetric efficiency:** It is defined as the ratio of volume of water striking actually to runner to the volume of water supplied by jet.

$$\eta_v = \frac{Q_a}{Q}$$ varies between 0.97 to 0.99 for Pelton wheel

7. **Generator efficiency:** It is defined as the ratio of generator power to the shaft power.

$$\eta_g = \frac{G.P}{S.P}$$

Design aspects of Pelton Wheel/turbine

1. **Velocity of jet:**

$$V_1 = C_v \sqrt{2gH}$$

Where $C_v =$ Coefficient of velocity (varies between 0.98 to 0.99)

$H =$ Net head on the turbine.

2. **Velocity of wheel**

$$u = K_u \sqrt{2gH}$$

Where $K_u =$ Speed ratio (varies between 0.43 to 0.48)

3. **Angle of Deflection of the jet:** The angle of deflection of the jet through the buckets is taken as 165^o if no angle of deflection is given.

4. **Mean diameter of the wheel (D):** The mean diameter or the pitch diameter of the pelton wheel is given by

$$u = \frac{\pi DN}{60}$$

5. **Jet ratio (m):** It is defined as the ratio of pitch diameter of the pelton wheel to the diameter of the jet.

$m = \frac{D}{d}$ Varies between 11 and 16 for maximum hydraulic efficiency.

Normally, jet ratio is adopted as 12 in practice.

6. **Bucket dimension:**

$$B = 3 \ to \ 4d$$

$$L = 2 \ to \ 3d$$

$$T = 0.8 \ to \ 1.2d$$

7. **No of jets:**

$$N_{jet} = \frac{\textbf{\textit{Required Discharge OR Total Discharge}}}{\textbf{\textit{Discharge through single jet}}}$$

8. **No of Buckets:**

$$Z = 15 + \frac{D}{2d} = 15 + 0.5m$$