Heat Transfer

Session – I

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Course contents: Heat Transfer

- **Course Code: 302042**

Course Objectives as described by SPPU

1. Identify the important modes of heat transfer and their applications.
2. Formulate and apply the general three dimensional heat conduction equations.
3. Analyze the thermal systems with internal heat generation and lumped heat capacitance.
4. Understand the mechanism of convective heat transfer
5. Determine the radiative heat transfer between surfaces.
6. Describe the various two phase heat transfer phenomenon. Execute the effectiveness and rating of heat exchangers.

Additional course Objective we will have

To understand, formulate and apply the generalized energy equation for heat transfer

Course contents

- **Please refer SPPU curriculum**

- **Text Books**

1. F.P. Incropera, D.P. Dewitt, Fundamentals of Heat and Mass Transfer, John Wiley.
2. Y. A. Cengel and A.J. Ghajar, Heat and Mass Transfer – Fundamentals and Applications, Tata McGraw Hill Education Private Limited.
3. S.P. Sukhatme, A Textbook on Heat Transfer, Universities Press.
4. R.C. Sachdeva, Fundamentals of Engineering Heat and Mass Transfer, New Age Science.
5. P.K. Nag, Heat & Mass Transfer, McGraw Hill Education Private Limited.
6. M. M. Rathod, Engineering Heat and Mass Transfer, Third Edition, Laxmi Publications, New Delhi
7. V. M. Domkundwar, Heat Transfer,

Course Outcomes

CO 1: Analyze the various modes of heat transfer and implement the basic heat conduction equations for steady one dimensional thermal system.

CO 2: Implement the general heat conduction equation to thermal systems with and without internal heat generation and transient heat conduction.

CO 3: Analyze the heat transfer rate in natural and forced convection and evaluate through experimentation investigation.

CO 4: Interpret heat transfer by radiation between objects with simple geometries.

CO 5: Analyze the heat transfer equipment and investigate the performance.

Additional course Outcome we will have

Students will be able to apply generalized energy equation to formulate the heat transfer problem

General Introduction

Three modes of Heat Transfer: Conduction, Convection and Radiation

Conduction:

- Transfer of energy from more energetic particles to less energetic particles
- Solids: energy transfer due to lattice vibrations/waves
- Fourier's law:

$$q = -k \frac{dT}{dx}$$

General Introduction

Convection:

- Transfer of energy due to bulk fluid motion in addition to random molecular motion

- Newton's law of cooling:

$$q = h(T_s - T_b)$$

General Introduction

Radiation:

- Transfer of energy due electromagnetic waves
- Stefan Boltzmann Law

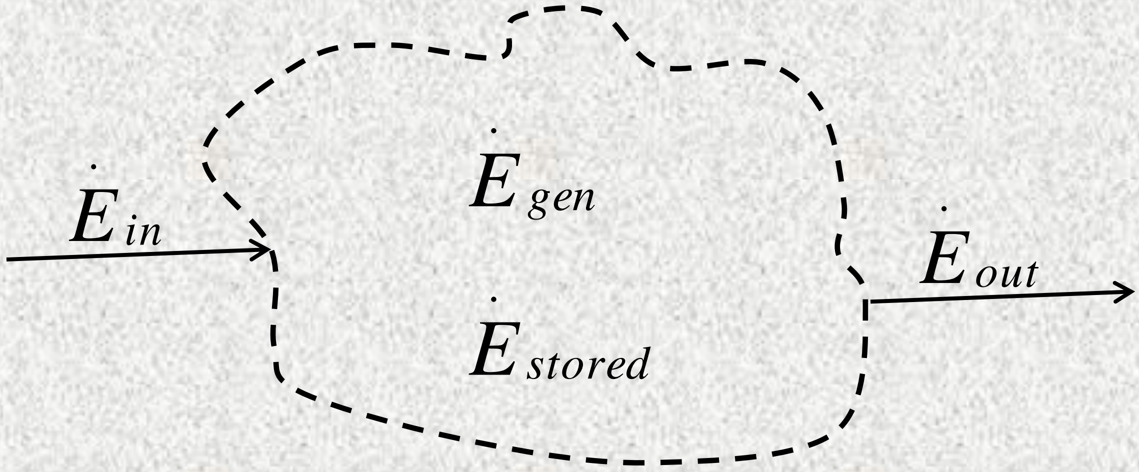
$$q = \varepsilon\sigma(T_s^4 - T_{surr}^4)$$

Emissivity

5.68 × 10^{-8} W/m^2k^4

General Introduction

Conservation of energy for a control volume:



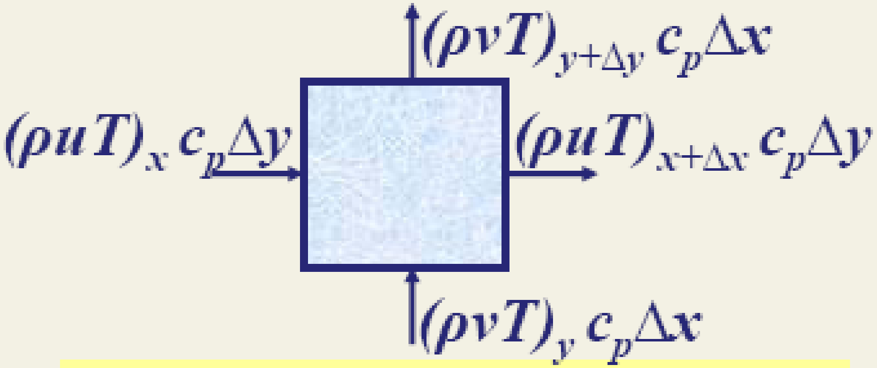
$$\dot{E}_{in} + \dot{E}_{gen} - \dot{E}_{out} = \dot{E}_{stored}$$

$$\dot{E}_{stored} = 0$$ ⇒ Steady state

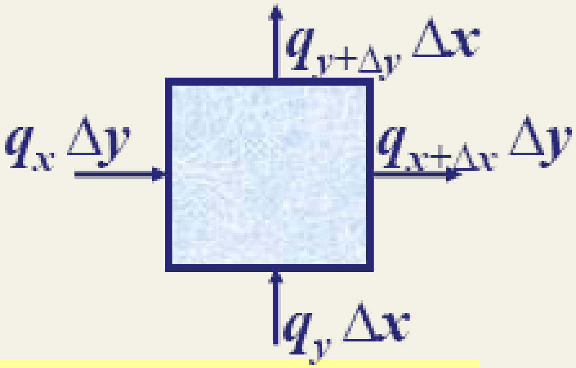
Derivation of Energy Equation

- **LAW OF CONSERVATION OF ENERGY:**

Rate of change of Internal Energy of the fluid inside the CV
= Rate at which Internal Energy (Inflow-Outflow)
+ Rate of Conduction Heat Transfer to the Fluid
+ Heat Generation Rate



Internal Energy Inflow and outflow at the boundary



Conduction Heat Transfer Rate

Derivation of Energy Equation

- Rate of change of Internal Energy of the fluid inside the CV = $\frac{\partial}{\partial t}(\rho \Delta x \Delta y c_p T)$

- Internal Energy Rate at (Inflow-Outflow)

$$\Rightarrow -\Delta x \Delta y c_p \left[\frac{\partial}{\partial x}(\rho u T) + \frac{\partial}{\partial y}(\rho v T) \right]$$

$$\Rightarrow -\Delta x \Delta y c_p \nabla.(\rho \vec{u} T)$$

- Heat Generation Rate: $\overline{q} \Delta x \Delta y$

Derivation of Energy Equation

- **Conduction Heat Transfer Rate to the Fluid**

$$\Delta x \Delta y \left[\lim_{\Delta x \to 0} \frac{q_x - q_{x+\Delta x}}{\Delta x} + \lim_{\Delta y \to 0} \frac{q_y - q_{y+\Delta y}}{\Delta y} \right]$$

$$\Rightarrow -\Delta x \Delta y \left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} \right) = \Delta x \Delta y k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial x^2} \right)$$

- **Constitutive Relationship:** $\vec{q} = -k \nabla T$
- **Energy Equation**

$$\frac{\partial(\rho T)}{\partial t} + \nabla.(\rho \vec{u} T) = \frac{k}{c_p} \nabla^2 T + \overline{q}$$

Governing Equations: 2-D Navier-Stokes

- **Continuity:** $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$
- **X-Momentum:**

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho u u)}{\partial x}+\frac{\partial(\rho v u)}{\partial y}=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

- **Y-Momentum:**

$$\frac{\partial(\rho v)}{\partial t}+\frac{\partial(\rho u v)}{\partial x}+\frac{\partial(\rho v v)}{\partial y}=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

Governing Equations: 2-D Navier-Stokes

- **Energy**

$$\frac{\partial T}{\partial t}+\frac{\partial(\rho u T)}{\partial x}+\frac{\partial(\rho v T)}{\partial y}=\frac{k}{c_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\bar{q}$$

- **General**

$$\frac{\partial \phi}{\partial t}+\frac{\partial(\rho u \phi)}{\partial x}+\frac{\partial(\rho v \phi)}{\partial y}=\Gamma_\phi\left(\frac{\partial^2 \phi}{\partial X^2}+\frac{\partial^2 \phi}{\partial Y^2}\right)+S_\phi$$

Unsteady Convection Diffusion Source

where $\phi = u,\ v,\ T$ and

$$S_\phi = -\partial p/\partial x, -\partial p/\partial y, \bar{q};\ \text{respectively.}$$

$\Gamma_\phi = \mu$ for mom. and $\left(k/c_p\right)$ for energy equation.

Non-Dimensional Governing Equation

- Dimensional G.E

$$\nabla \cdot (\rho \vec{u}) = 0$$

$$\frac{\partial (\rho \vec{u})}{\partial t} + \nabla . (\rho \vec{u} \vec{u}) = -\nabla p + \mu \nabla^2 \vec{u} + \vec{X}$$

$$\frac{\partial (\rho T)}{\partial t} + \nabla . (\rho \vec{u} T) = \frac{k}{c_p} \nabla^2 T + \overline{q}$$

- Non-Dimensional Parameters

$$X = \frac{x}{L_c}, \quad Y = \frac{Y}{L_c}, \quad \tau = \frac{t}{L_c / u_c}, \quad U = \frac{u}{u_c},$$

$$V = \frac{v}{u_c}, \quad P = \frac{p}{\rho u_c^2}, \quad \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)} = \frac{(T - T_\infty)}{(q_w L_c / k)}$$

Non-Dimensional Governing Equation

- Continuity

$$\nabla \cdot \vec{U} = 0$$

- Momentum

$$\frac{\partial \vec{U}}{\partial \tau} + \nabla \cdot \left(\vec{U}\vec{U} \right) = -\nabla P + \frac{1}{\mathrm{Re}} \nabla^2 \vec{U} + \vec{X}^*$$

- Energy

$$\frac{\partial \theta}{\partial \tau} + \nabla \cdot \left(\vec{U}\theta \right) = \frac{1}{(\mathrm{Re}\,\mathrm{Pr})} \nabla^2 \theta + \bar{q}^*$$

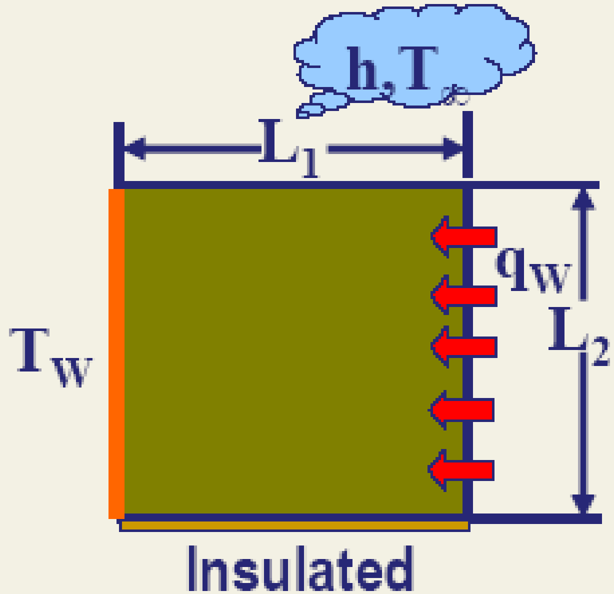
Initial and Boundary Conditions

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{\bar{q}}{\rho c_p}$$

1 I.C (for $\partial T/\partial t$) 2 B.C.s (for $\partial^2 T/\partial x^2$) 2 B.C.s (for $\partial^2 T/\partial y^2$)

Initial Condition:-

At $t = 0$, $T(x,y,z)=T_0$



Boundary Conditions:-

1. Dirichlet (Left Wall) :- $T = T_w$
2. Neuman :- dT/dn =constant (c)

   Bottom Wall: $c=0$;

   Right Wall: $c=- q_W/k$
3. Robin or mixed (Top Wall):-

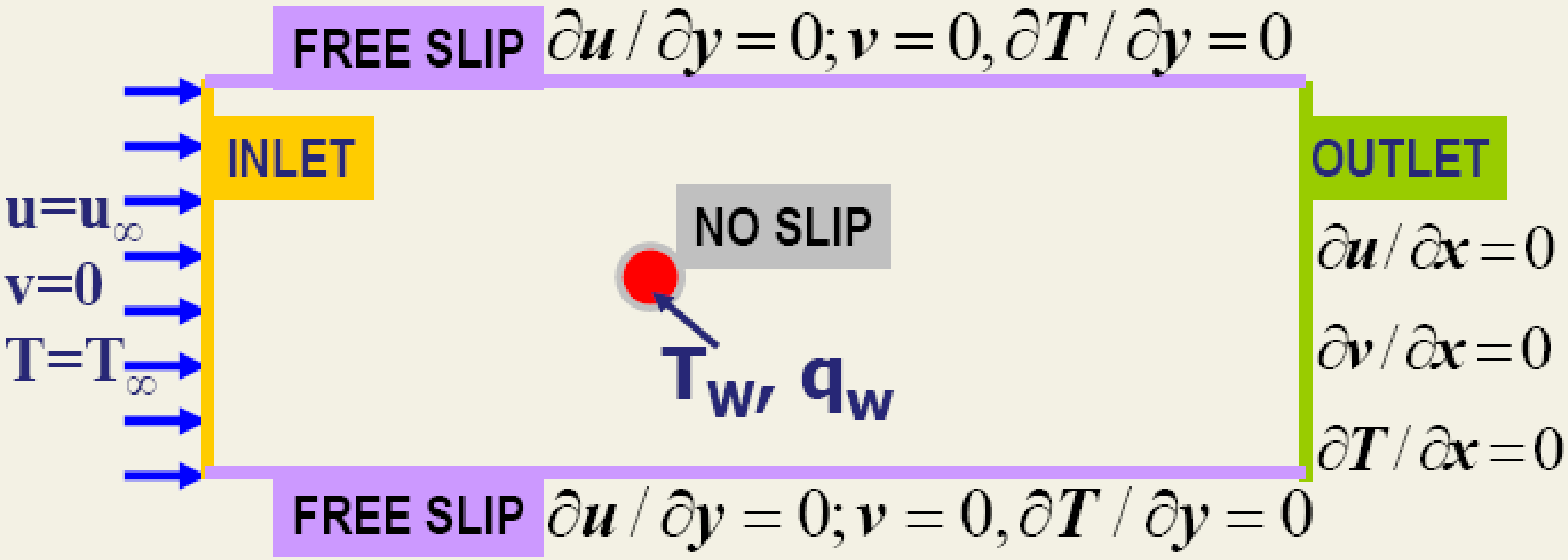
$$aT + b\frac{dT}{dn} = const$$

$$q_{cond} = q_{conv} \Rightarrow -k\,dT/dy = h(T - T_\infty)$$

$$hT + k\frac{dT}{dy} = hT_\infty$$

Initial and Boundary Conditions

- Free Stream Flow and Heat Transfer across a Cylinder
  - Initial Condition: $u=0$; $v=0$ and $T=T_\infty$
  - Boundary Conditions:



Properties

- Fluid Property
  - Density
  - Dynamic Viscosity
  - Kinematic Viscosity
  - Specific Heat
  - Coefficient of thermal Conductivity
  - Thermal Diffusivity
- Flow Property
  - Velocity
  - Pressure
  - Temperature
  - Vorticity

Non-dimensional Parameters for Flow and Heat Transfer

- Governing Parameters
  - Reynolds Number
  - Weber Number
  - Fraude Number
  - Grashof Number
  - Prandtl Number
- Engineering Parameters
  - Skin Friction Coefficient
  - Friction Factor
  - Drag Coefficient
  - Lift Coefficient
  - Strouhal Number
  - Nusselt Number

Mathematical Character of PDE

- Second order

$$A\frac{\partial^2 \phi}{\partial^2 x}+B\frac{\partial^2 \phi}{\partial x \partial y}+C\frac{\partial^2 \phi}{\partial^2 y}+D\frac{\partial \phi}{\partial x}+E\frac{\partial \phi}{\partial y}+F\phi = G(x,y)$$

  - Homogeneous ($G=0$) and Non-Homogeneous
  - Linear, Quasilinear and Nonlinear
  - Elliptic ($B^2-4AC<0$, Equilibrium Prob.)
  - Parabolic ($B^2-4AC=0$, Marching Prob.)
  - Hyperbolic ($B^2-4AC>0$, Marching Prob.)
- Navier-Stokes Equations
  - Combined Elliptic-Parabolic Nonlinear Equations (Initial-Boundary value Prob.)

Classification of Governing Equations

- **Elliptic Equation: Boundary Value Prob.**

$$\left.\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}=0\right\}\begin{array}{l} A=C=1; B=0 \\ \Rightarrow B^2-4AC<0 \end{array}$$

- **Parabolic Equation: Initial-Boundary-Value Prob.**

$$\left.\frac{\partial \phi}{\partial t}=\alpha\frac{\partial^2 \phi}{\partial x^2}\right\}\begin{array}{l} A=E=1; B=C=0 \\ \Rightarrow B^2-4AC=0 \end{array}$$

- **Hyperbolic Equation: Initial-Boundary-Value Prob.**

$$\left.\frac{\partial^2 \phi}{\partial t^2}=c^2\frac{\partial^2 \phi}{\partial x^2}\right\}\begin{array}{l} A=1; B=0; C=-c^2 \\ \Rightarrow B^2-4AC=c^2>0 \end{array}$$

THE THERMAL PROPERTIES OF MATTER

THERMAL CONDUCTIVITY

- Thermal conductivity of a material is defined as the rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.
- The thermal conductivity of a material is a measure of how fast heat will flow in that material.
- A large value for thermal conductivity indicates that the material is a good heat conductor,
- A low value indicates that the material is a poor heat conductor or insulator.

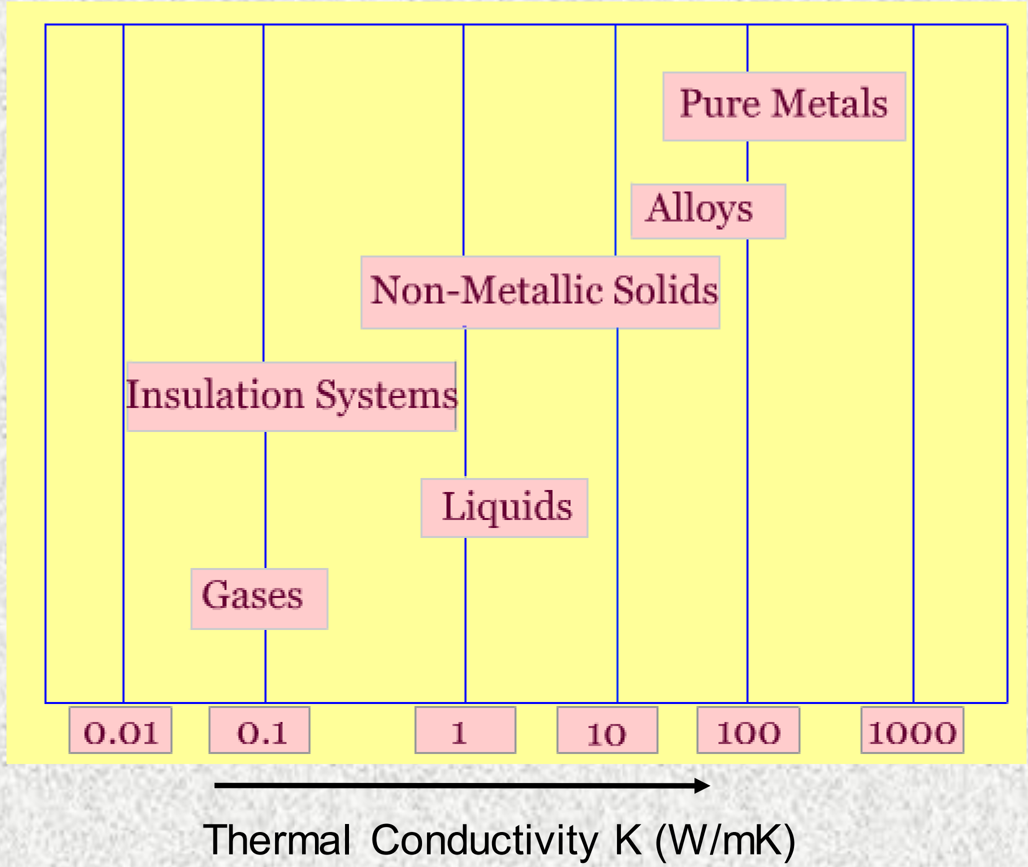


Figure: Range of thermal conductivity for various states of matter at normal temperature and pressure

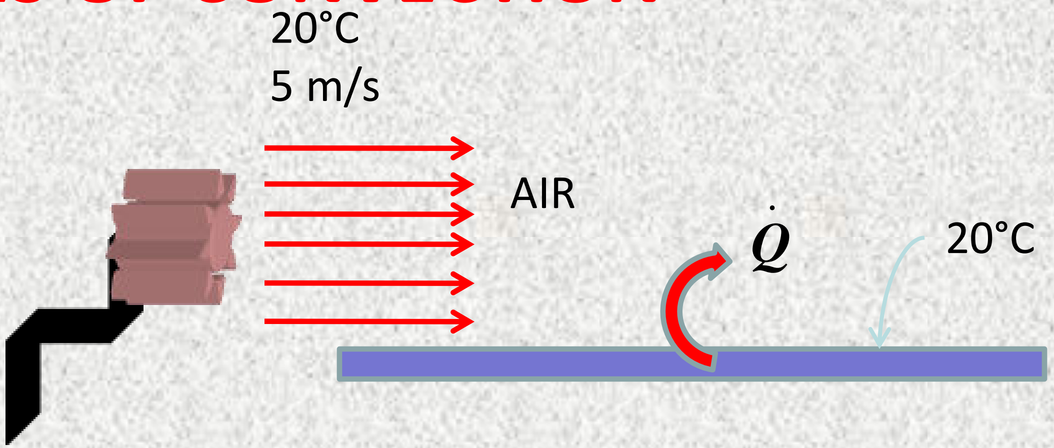
- Note that the thermal conductivity of a solid may be more than four orders of magnitude larger than that of a gas.
- This trend is largely due to differences in intermolecular spacing for the two states.

TYPICAL VALUES OF h

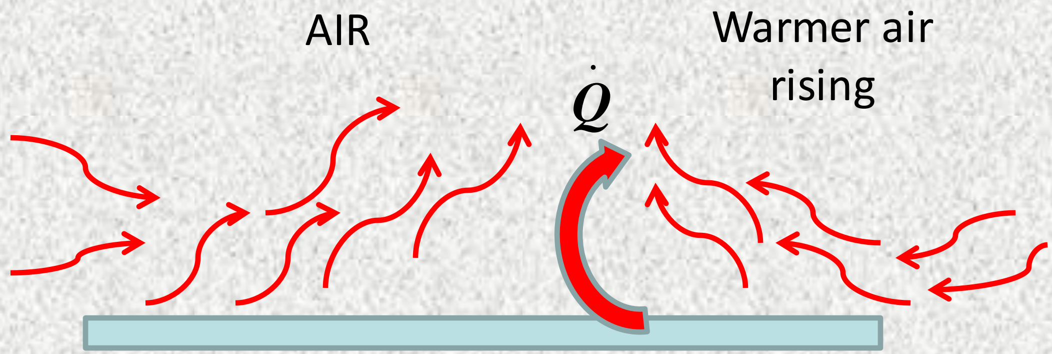
Process	h (W/m^2.K)
Free convection	
Gases	2-25
Liquids	50-1000
Forced Convection	
Gases	25-250
Liquids	50-20000
Boiling and condensation	2500-1,00,000

TYPES OF CONVECTION

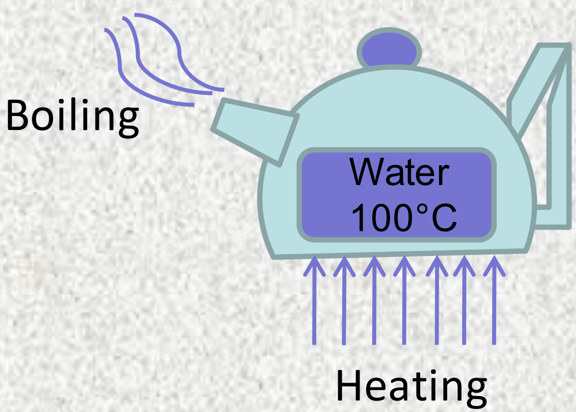
FORCED CONVECTION

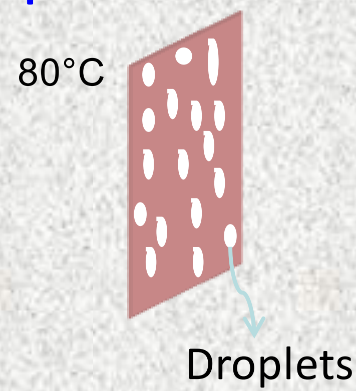


NATURAL CONVECTION



BOILING AND CONDENSATION – involve phase change

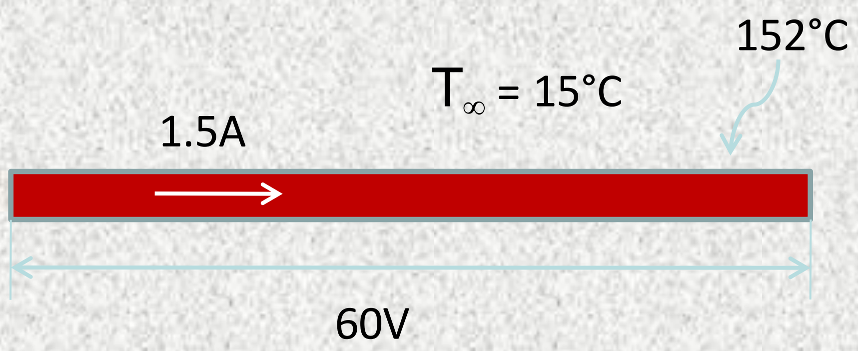




Problem: A 2 m long, 0.3 cm diameter electrical wire extends across a room at 15^0 C as given in schematic. Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 152^0 C in steady operation. Also, the voltage drop and electric current through the wire are measured to be 60 V and 1.5 A, respectively. Disregarding any heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.

Known: wire dimensions, room temperature, surface temperature of the wire, voltage drop and electric current through the wire.

Find: convection heat transfer coefficient between the outer surface of the wire and the air in the room.



Assumptions:

- Steady operating conditions exist since the temperature readings do not change with time
- Radiation heat transfer is negligible.

Analysis

When steady operating conditions are reached, the rate of heat loss from the wire will equal the rate of heat generation in the wire as a result of resistance heating.

$$\dot{Q} = \dot{E}_{generated} = VI = 60(1.5) = 90\ W$$

That is , the surface area of the wire is

$$A = \pi D L = \pi (0.003)(2) = 0.01885\ m^2$$

Newton's law of cooling for convection heat transfer is expressed as

$$\dot{Q} = hA(T_s - T_\infty)$$

Disregarding any heat transfer by radiation and thus assuming all the heat loss from the wire to occur by convection, the convection heat transfer coefficient is to be determined to be

$$h = \frac{\dot{Q}}{A(Ts - T_\infty)} = \frac{90}{(0.01885)(152-15)} = 34.9\ W/m^2.C$$

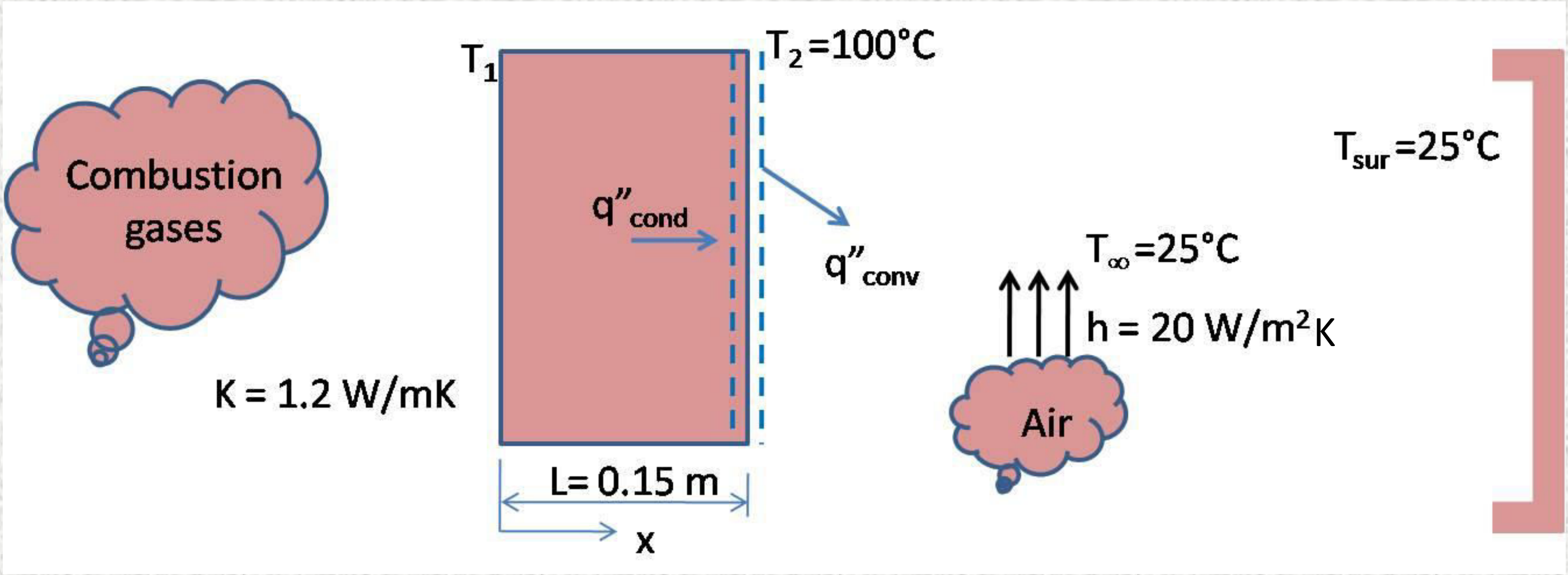
Comments:

Note that the simple setup described above can be used to determine the average heat transfer coefficients from a variety of surfaces in air. Also, heat transfer by radiation can be eliminated by keeping the surrounding surfaces at the temperature of the wire.

Problem: The hot combustion gases of a furnace are separated from the ambient air and its surroundings, which are at 25^0 C, by a brick wall 0.15 m thick. The brick has a thermal conductivity of 1.2 W/m.K. Under steady state conditions an outer surface temperature of 100^0 C is measured. Free convection heat transfer to the air adjoining the surface is characterized by a convection coefficient of h = 20 $W/m^2.K$. What is the brick inner surface temperature. Neglect any heat transfer by radiation.

Known: outer surface temperature of a furnace wall of prescribed thickness, thermal conductivity, ambient conditions

Find: Wall inner surface temperature



Assumptions:

1. Steady state conditions
2. One dimensional heat transfer by conduction across the wall
3. Radiation heat transfer is neglected

Analysis:

The inside surface temperature may be obtained by performing an energy balance at the outer surface.

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

it follows that, on a unit area basis,

$$q''_{cond} - q''_{conv} = 0 \Rightarrow k\frac{T_1 - T_2}{L} = h\left(T_2 - T_\infty\right)$$

$$1.2\frac{T_1 - 100}{0.15} = 20\left(100 - 25\right) \Rightarrow T_1 = 287.5^o C$$

Comments

Brick surface temperature is high

Heat Conduction

General heat diffusion equation in coordinate free form:

$$\rho C_p \left(\frac{\partial T}{\partial t} + \nabla . \vec{u} T \right) = k \nabla^2 T + \bar{q}$$

Heat diffusion equation in Cartesian coordinates:

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$$

Heat diffusion equation in Cartesian coordinates if thermal conductivity is constant:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Heat Conduction

Heat diffusion equation in Cartesian coordinates if heat transfer is steady state:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=0$$

If the heat transfer is one dimensional (e.g., in the *x* direction) and there is no energy generation , above equation reduces to

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)=0$$

The most important implication of this result is that under steady state, one dimensional conditions with no energy generation, the heat flux is a constant in the direction of heat transfer

BOUNDARY AND INITIAL CONDITIONS

- To determine the temperature distribution in a medium, it is necessary to solve the appropriate form of the heat equation.

- However, such a solution depends on the physical conditions existing at the boundaries of the medium and, if the situation is time dependent, on conditions existing in the medium at some initial time.

- Because the heat equation is second order in the spatial coordinates, two boundary conditions must be expressed for each coordinate to describe the system.

- Because the equation is first order in time, however, only one condition, termed the initial condition, must be specified.

The three kinds of boundary conditions commonly encountered in heat transfer are summarized in Table 2.1.

- The conditions are specified at the surface $x = 0$ for a one-dimensional system.

- Heat transfer is in the positive x direction with the temperature distribution, which may be time dependent, designated as $T(x, t)$.

- The first condition corresponds to a situation for which the surface is maintained at a fixed temperature T_s. It is commonly termed a Dirichlet condition, or a boundary condition of the first kind.

- Example: when the surface is in contact with a melting solid or a boiling liquid. In both cases there is heat transfer at the surface, while the surface remains at the temperature of the phase change process.

- The second condition corresponds to the existence of a fixed or constant heat flux q_s'' at the surface. This heat flux is related to the temperature gradient at the surface by Fourier's Law, which may be expressed as

$$q_x'' = -k \left. \frac{\partial T}{\partial x} \right|_{x=0} \qquad (2.21)$$

- The second condition is termed as Neumann condition, or a boundary condition of the second kind, and may be realized by bonding a thin film or patch electric heater to the surface.

- A special case of this condition corresponds to the perfectly insulated, or adiabatic, surface for which

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = 0$$

- The boundary condition of the third kind corresponds to the existence of convection heating (or cooling) at the surface and is obtained from the surface energy balance.

ONE-DIMENSIONAL STEADY STATE CONDUCTION

- In a **one-dimensional** system, **temperature gradients** exist along a **single coordinate direction**, and heat transfer occurs exclusively in that direction.
- The system is characterized by **steady state conditions** if the **temperature** at each point is **independent of time**.

THE PLANE WALL

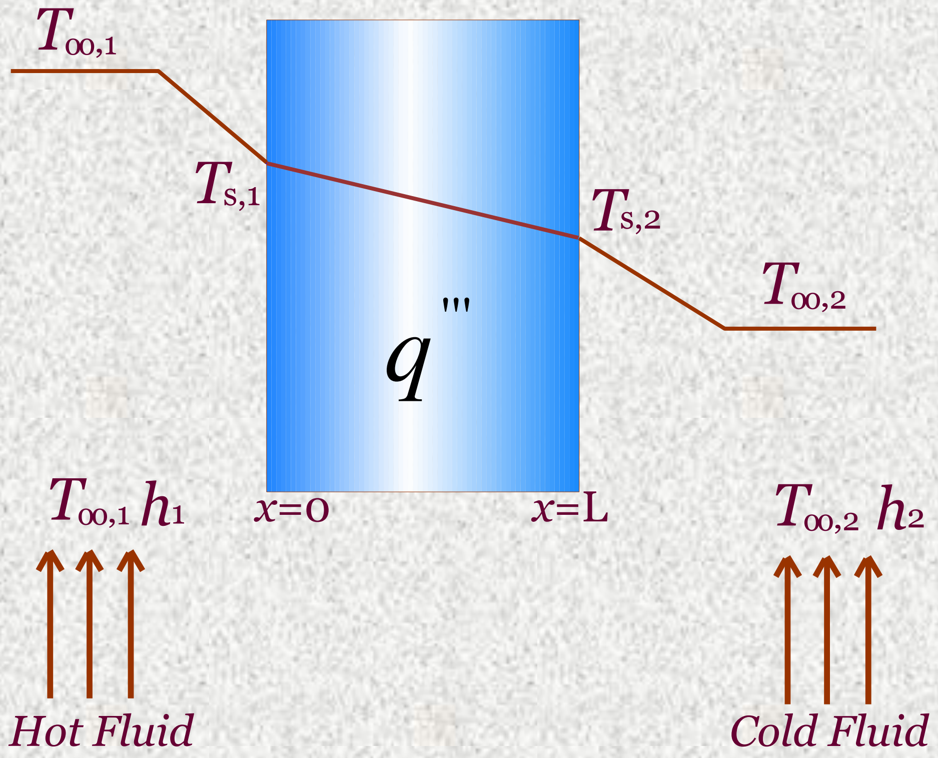
For one dimensional conduction in a plane wall, temperature is a function of the *x* coordinate only and heat is transferred exclusively in this direction.

In Figure 2.5, a plane wall separates two fluids of different temperatures.

Heat transfer occurs,

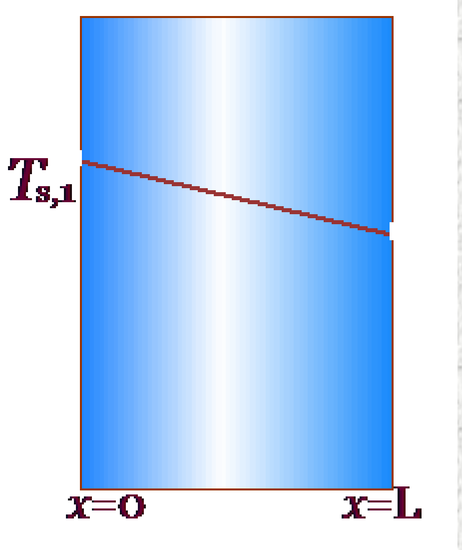
- by convection from the hot fluid at $T_{\infty,1}$ to one surface of the wall at $T_{s,1}$
- by conduction through the wall, and
- by convection from the other surface of the wall at $T_{s,2}$ to the cold fluid at $T_{\infty,2}$

ONE DIMENSIONAL STEADY STATE HEAT TRANSFER IN PLANE WALL



$$\nabla . k \nabla T + q''' = \frac{\partial}{\partial t}(\rho C_p T)$$

ONE DIMENSIONAL STEADY STATE CONDUCTION HEAT TRANSFER IN PLANE WALL



$$\nabla . k \nabla T + q''' = \frac{\partial}{\partial t}(\rho C_p T)$$

No heat generation steady

$$\frac{d}{dx}\left(k \frac{\partial T}{\partial x}\right) = 0 \Rightarrow T(x) = C_1 x + C_2$$

Temperature distribution along x direction is

$$T(x) = \left(T_{s,2} - T_{s,1}\right)\frac{x}{L} + T_{s,1}$$

ONE DIMENSIONAL STEADY STATE CONDUCTION HEAT TRANSFER IN PLANE WALL

Steady state heat transfer rate along x direction:

$$q_x = -kA\frac{dT}{dx} = \frac{kA}{L}\left(T_{s,1} - T_{s,2}\right)$$

Steady state heat flux along x direction:

$$q_x'' = \frac{q_x}{A} = \frac{k}{A}\left(T_{s,1} - T_{s,2}\right)$$

Note that both the heat rate q_x and heat flux q''_x are constants, independent of x.

THERMAL RESISTANCE FOR HEAT TRANSFER IN PLANE WALL

- There exists an analogy between the diffusion of heat and electrical charge.

- Thermal resistance may be associated with the conduction of heat in the same fashion as an electrical resistance is associated with the conduction of electricity.

- Defining resistance as the ratio of a driving potential to the corresponding transfer rate IT follows that the thermal resistance for conduction is

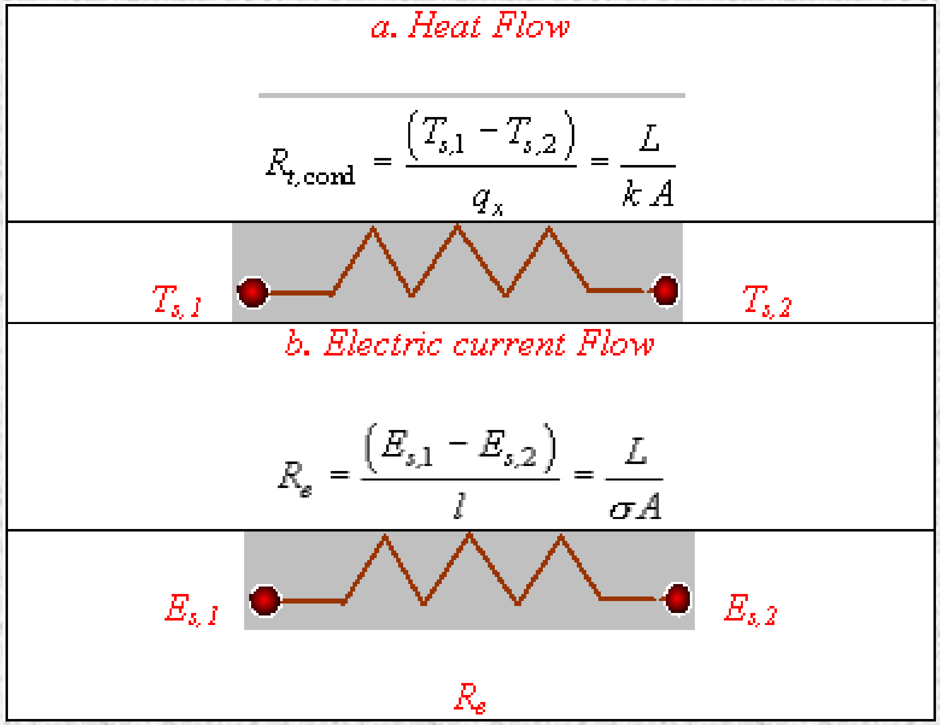
$$R_{t,\text{cond}} = \frac{\left(T_{s,1} - T_{s,2}\right)}{q_x} = \frac{L}{kA}$$

- Similarly, for electrical conduction, Ohm's law provides an electrical resistance of the form

$$R_e = \frac{\left(E_{s,1} - E_{s,2}\right)}{I} = \frac{L}{\sigma A}$$

THERMAL RESISTANCE FOR HEAT TRANSFER IN PLANE WALL: AN ELECTRICAL ANALOGY

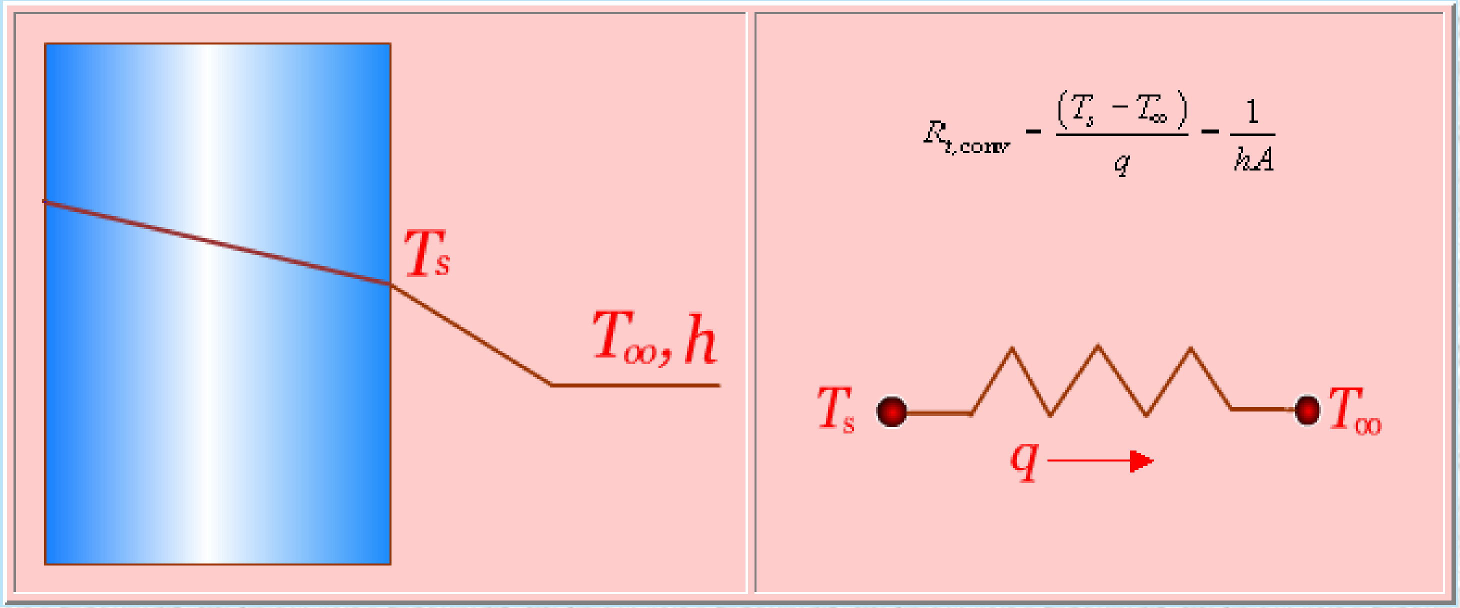
Thermal resistance for conduction



$$R_{t,\text{cond}} = \frac{\left(T_{s,1} - T_{s,2}\right)}{q_x} = \frac{L}{kA}$$

THERMAL RESISTANCE FOR HEAT TRANSFER IN PLANE WALL: AN ELECTRICAL ANALOGY

Thermal resistance for convection

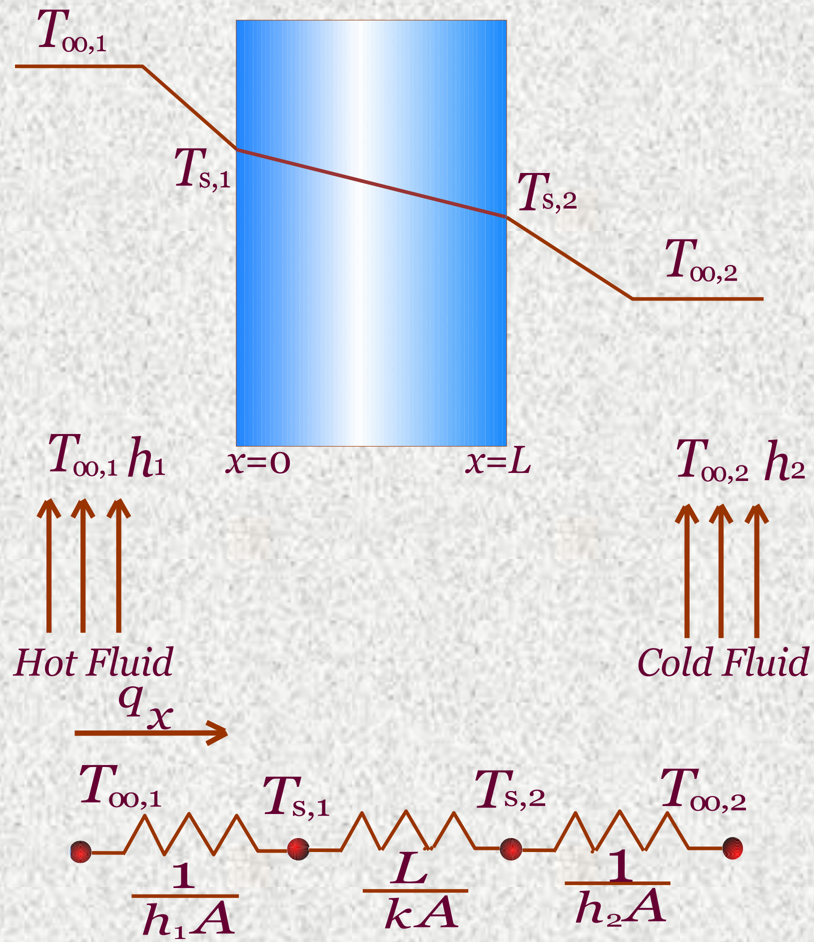


$$q = hA\left(T_s - T_\infty\right)$$

$$R_{t,\text{conv}} = \frac{\left(T_s - T_\infty\right)}{q} = \frac{1}{hA}$$

THERMAL RESISTANCE FOR HEAT TRANSFER IN PLANE WALL: AN ELECTRICAL ANALOGY

Thermal resistance network for heat transfer through a plane wall



THERMAL RESISTANCE FOR HEAT TRANSFER IN PLANE WALL: AN ELECTRICAL ANALOGY

Under steady state conditions, we have

$$\begin{pmatrix} \text{Rate of Heat Convection} \\ \text{int o the Wall} \end{pmatrix} = \begin{pmatrix} \text{Rate of Heat Conduction} \\ \text{through the Wall} \end{pmatrix} = \begin{pmatrix} \text{Rate of Heat Convection} \\ \text{from the Wall} \end{pmatrix}$$

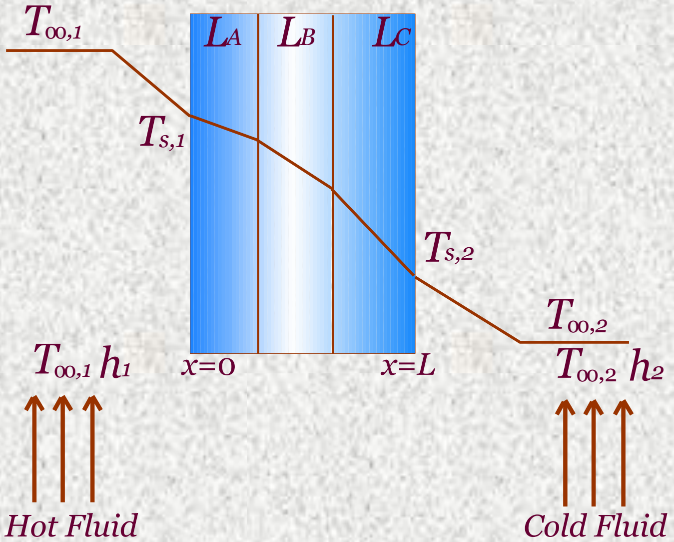
$$q_x = h_1 A \left(T_{\infty,1} - T_{s,1}\right) = \left(T_{s,1} - T_{s,2}\right)\frac{kA}{L} = h_2 A \left(T_{s,2} - T_{\infty,2}\right)$$

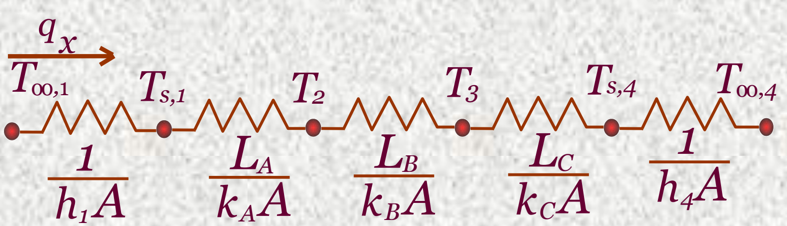
$$q_x = \frac{\left(T_{\infty,1} - T_{s,1}\right)}{1/h_1 A} = \frac{\left(T_{s,1} - T_{s,2}\right)}{L/kA} = \frac{\left(T_{s,2} - T_{\infty,2}\right)}{1/h_2 A}$$

In terms of overall temperature difference $q_x = \dfrac{\left(T_{\infty,1} - T_{\infty,2}\right)}{R_{tot}}$

In general $q = \dfrac{\left(T_i - T_j\right)}{\sum \text{All resistances between the driving } \Delta T \text{ , } T_i - T_j}$

THERMAL RESISTANCE FOR HEAT TRANSFER IN COMPOSITE WALL: AN ELECTRICAL ANALOGY





$$q_x = \frac{\left(T_{\infty,1} - T_{\infty,4}\right)}{\sum R_t}$$

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{\left(1/h_1 A\right)} = \frac{T_{s,1} - T_2}{\left(L_A/k_A A\right)} = \frac{T_2 - T_3}{\left(L_B/k_B A\right)} = \ldots.$$

In terms of overall heat transfer coefficient

$$q_x = UA\,\Delta T$$

$$U = \frac{1}{R_{tot}\ A} = \frac{1}{\left[\left(1/h_1\right) + \left(L_A/k_A\right) + \left(L_B/k_B\right) + \left(L_C/k_C\right) + \left(1/h_4\right)\right]}$$

In general

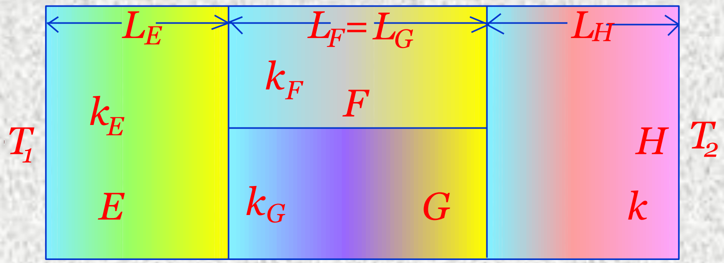
$$R_{tot} = \sum R_{tot} = \frac{\Delta T}{q} = \frac{1}{UA}$$

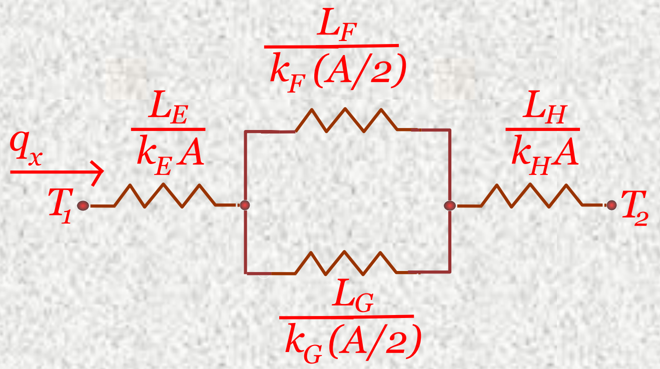
THERMAL RESISTANCE FOR HEAT TRANSFER IN COMPOSITE WALL: AN ELECTRICAL ANALOGY

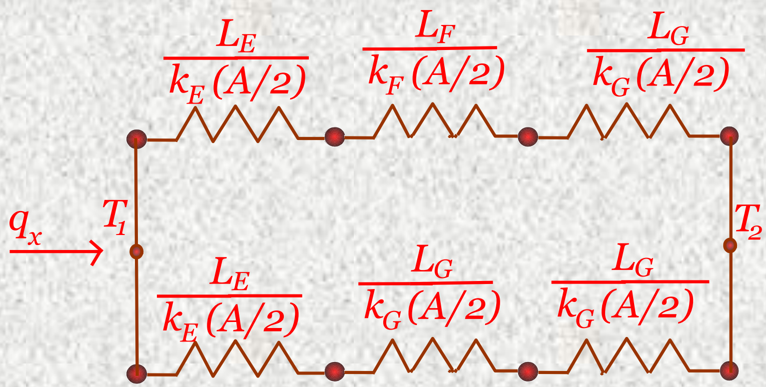
Composite walls may also be characterized by series-parallel configurations

$$q = q_1 + q_2$$

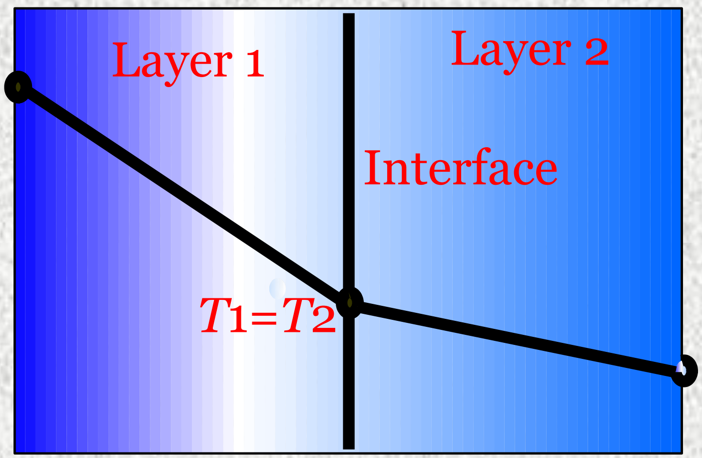
$$= \frac{\Delta T}{\left(L_f / k_f (A/2) \right)} = \frac{\Delta T}{\left(L_g / k_g (A/2) \right)}$$



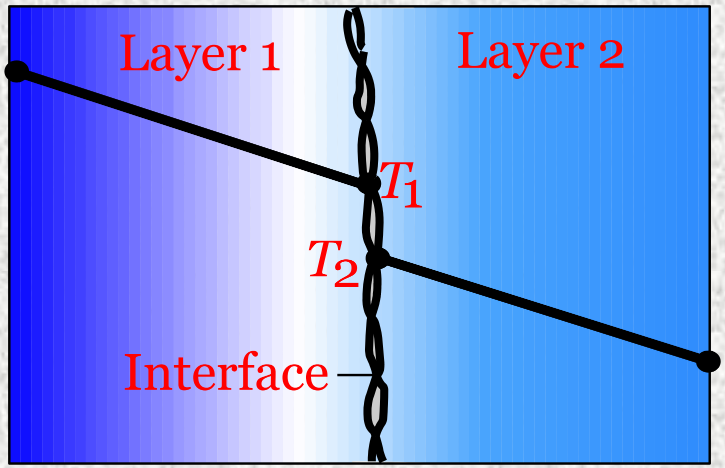




Contact Thermal Resistance



Ideal Thermal Contact



Actual Thermal Contact

An interface will contain numerous air gaps of varying sizes that act as insulation because of the low thermal conductivity of air.

Thus, an interface offers some resistance to heat transfer, and this resistance per unit interface area is called thermal contact resistance, $R''_{t,c}$ given by

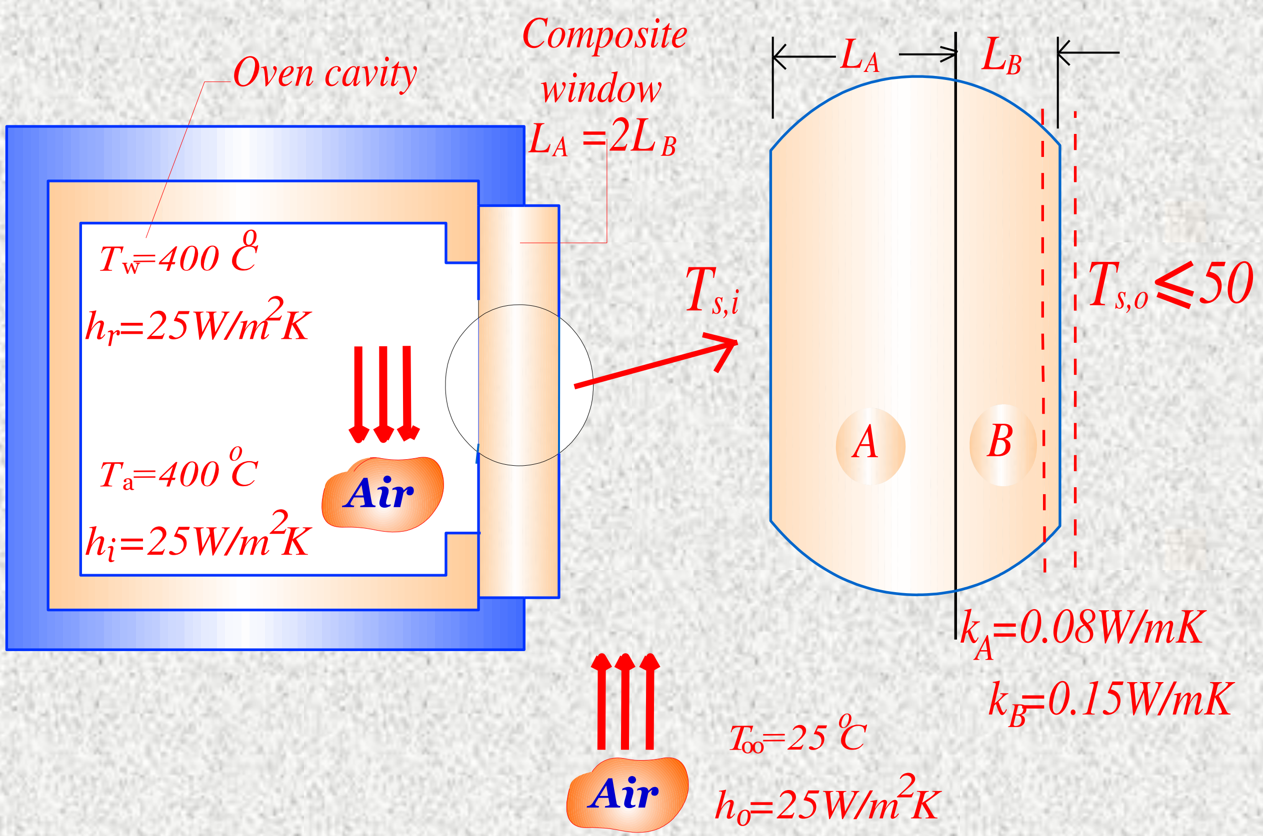
$$R''_{t,c} = \frac{T_1 - T_2}{q''_x}$$

- For solids whose thermal conductivities exceed that of the interfacial fluid, the contact resistance may be reduced by increasing the area of the contact spots.
- Such an increase may be effected by increasing the joint pressure and/or by reducing the roughness of the mating surfaces.
- The contact resistance may also be reduced by selecting an interfacial fluid of large thermal conductivity.
- In this respect, no fluid (an evacuated interface) eliminates conduction across the gap, thereby increasing the contact resistance.

PROBLEM 2.2

A leading manufacturer of household appliances is proposing a self-cleaning oven design that involves use of a composite window separating the oven cavity from the room air. The composite is to consist of two high temperature plastics (A and B) of thicknesses $L_A = 2L_B$ and thermal conductivities k_A=0.15 *W/m.K* and k_B= 0.08 *W/m.K*. During the self-cleaning process, the oven wall and air temperatures, T_w and T_a, are 400^0C, while the room air temperature T_∞ is 25^0C. The inside convection and radiation heat transfer coefficients h_i and h_r, as well as the outside convection coefficient h_o, are each approximately 25 $W/m^2.K$. What is the minimum window thickness, $L=L_A+L_B$, needed to ensure a temperature that is 50^0C or less at the outer surface of the window? This temperature must not be exceeded for safety reasons

Figure:



Known: The properties and relative dimensions of plastic materials used for a composite oven window, and conditions associated with self-cleaning operation

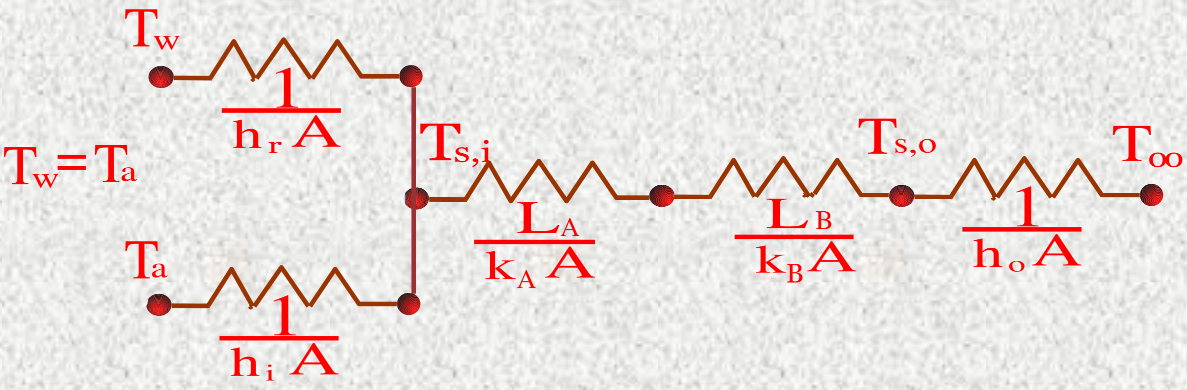
Find : Composite thickness $L_A + L_B$ needed to ensure safe operation

Assumptions:

- Steady state conditions exist
- Conduction through the window is one dimensional
- Contact resistance is negligible
- Radiation absorption within the window is negligible; hence no internal heat generation
- Radiation exchange between window outer surface and surroundings is negligible
- Each plastic is homogeneous with constant properties

Analysis:

The thermal circuit can be constructed by recognizing that resistance to heat flow is associated with convection at the outer surface, conduction in the plastics, and convection and radiation at the inner surface. Accordingly, the circuit and the resistances are of the following form:



Since the outer surface temperature of the window, $T_{s,o}$ is prescribed, the required window thickness may be obtained by applying an energy balance at this surface. That is, from Equation 1.7

$$\dot{E}_{in} = \dot{E}_{out}$$

where, with $T_w = T_a$,

$$\dot{E}_{in} = q = \frac{T_a - T_{s,o}}{\sum R_t}$$

and

$$\dot{E}_{out} = q = h_o A \left(T_{s,o} - T_\infty \right)$$

The total thermal resistance between the oven cavity and the outer surface of the window includes an effective resistance associated with convection and radiation, which act in parallel at the inner surface of the window, and the conduction resistances of the window materials. Hence,

$$\sum R_t = \left(\frac{1}{1/h_i A} + \frac{1}{1/h_r A} \right)^{-1} + \frac{L_A}{k_A A} + \frac{L_B}{k_B A}$$

or

$$\sum R_t = \frac{1}{A}\left(\frac{1}{h_i + h_r} + \frac{L_A}{k_A} + \frac{L_A}{2k_B}\right)$$

substituting into the energy balance, it follows that

$$\frac{T_a - T_{s,o}}{\left(h_i + h_r\right)^{-1} + \left(\frac{L_A}{k_A}\right) + \left(\frac{L_A}{2k_B}\right)} = h_o\left(T_{s,o} - T_\infty\right)$$

hence for solving L_A,

$$L_A = \frac{\left[\left(1/h_o\right)\left(T_a - T_{s,o}\right) \Big/ \left(T_{s,o} - T_\infty\right)\right] - \left(h_i + h_r\right)^{-1}}{\left(\frac{1}{k_A} + \frac{1}{2k_B}\right)}$$

$$L_A = \frac{0.04\left(\frac{400 - 50}{50 - 25}\right) - 0.02}{\left(1/0.15 + 1/0.16\right)} = 0.0418$$

since $L_B = L_A/2 = 0.0209$ m,

$L = L_A + L_B = 0.0627$ m = 62.7 mm

Comments:

1. The self cleaning operation is a transient process, as far as the thermal response of the window is concerned, and steady state conditions may not be reached in the time required for cleaning. However, the steady state condition provides the maximum possible value of $T_{s,o}$ and hence is well suited for the design calculation.

2. Radiation exchange between the oven walls and the composite window actually depends on the inner surface temperature $T_{s,1}$, and although it has been neglected, there is radiation exchange between the window and the surroundings, which depends on $T_{s,o}$.

A more complete analysis may be made to concurrently determine $T_{s,1}$ and $T_{s,o}$. Approximating the oven cavity as a large enclosure relative to the window and applying an energy balance, equation 1.12, at the inner surface it follows that

$$q''_{rad,i} + q''_{conv,i} = q''_{cond}$$

or

$$\varepsilon\sigma\left(T_{w,i}^4 - T_{s,i}^4\right) + h_i\left(T_a - T_{s,i}\right) = \frac{\left(T_{s,i} - T_{s,o}\right)}{\left(L_A/k_A\right) + \left(L_B/k_B\right)}$$

Approximating the kitchen walls as a large isothermal enclosure relative to the window, with $T_{w,o} = T_\infty$ and this time applying energy balance at the outer surface, it follows that

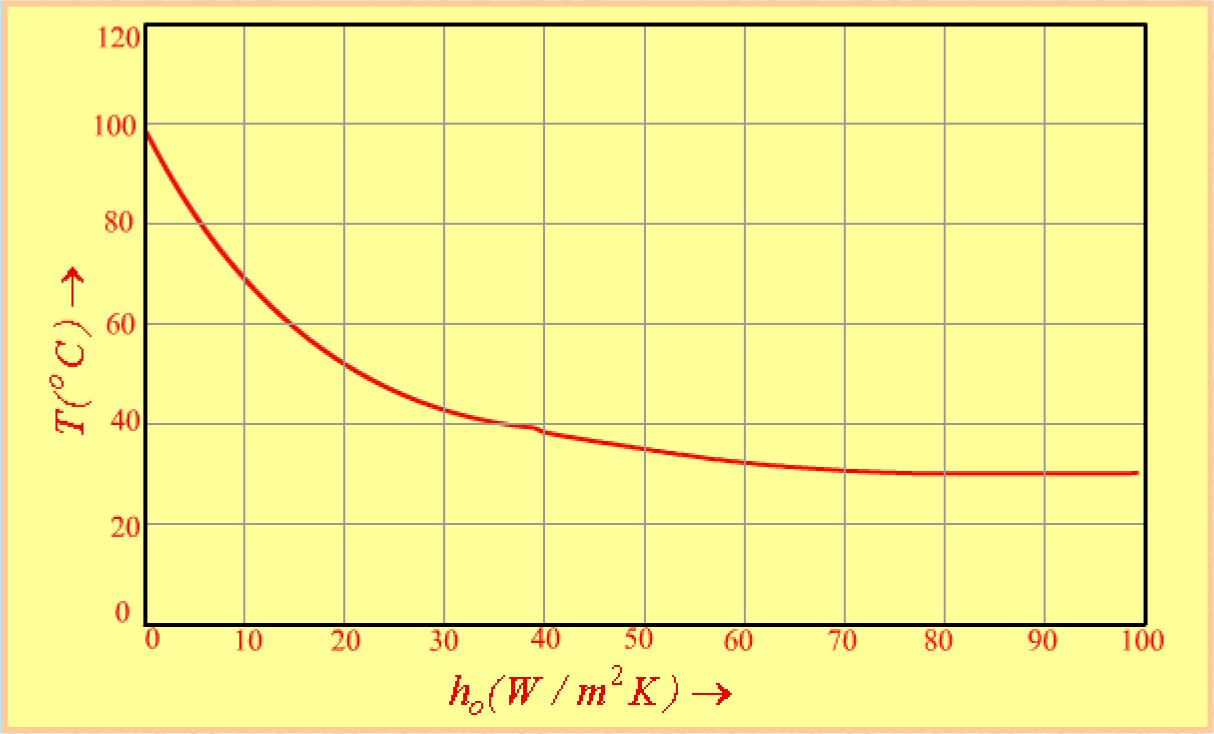
$$q''_{cond} = q''_{rad,o} + q''_{conv,o}$$

or

$$\frac{\left(T_{s,i} - T_{s,o}\right)}{\left(L_A/k_A\right) + \left(L_B/k_B\right)} = \varepsilon\sigma\left(T_{s,o}^4 - T_{w,o}^4\right) + h_o\left(T_{s,o} - T_\infty\right)$$

If all other quantities are known, Equations 1 and 2 may be solved for $T_{s,1}$ and $T_{s,0}$.

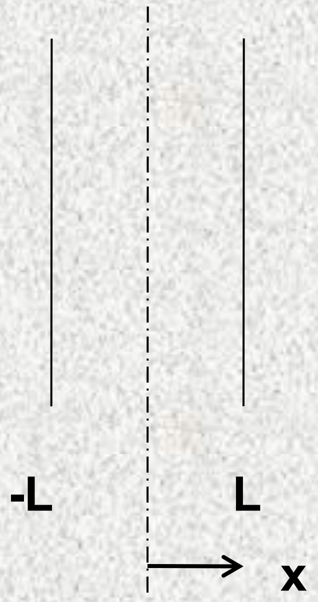
We wish to explore the effect on $T_{s,0}$ of varying velocity, and hence the convection coefficient, associated with airflow over the outer surface. With ε = 0.9 and all other conditions remaining the same, equations 1 and 2 have been solved for values of h_0 in the range $0 \le h_o \le 100$ $W/m^2\ K$ and the results are represented graphically.



3. Increasing h_0 reduces the corresponding convection resistance, and a value of h_0 =30 W/m^2 K would yield a safe to touch temperature of $T_{s,0}$ = 43°C . Because the conduction resistance is so large, the change h_0 in has a negligible effect on $T_{s,1}$. However it does influence the outer surface temperature, and as $h_o \rightarrow \infty$, $T_{s,o} \rightarrow \infty$

ONE DIMENSIONAL STEADY STATE CONDUCTION WITH HEAT GENERATION

$$\frac{\partial^2 T}{\partial x^2} + \frac{q'''}{k} = 0$$



$$T = \frac{-q'''}{k}\frac{x^2}{2} + C_1 x + C_2$$

Constant wall temperature BC: $T(-L) = T_1 \quad T(L) = T_2$

$$T(x) = \frac{-q'''}{k}\frac{L^2}{2}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_2 - T_1}{2}\frac{x}{L} + \frac{T_1 + T_2}{2}$$

Verify

ONE DIMENSIONAL STEADY STATE CONDUCTION WITH HEAT GENERATION

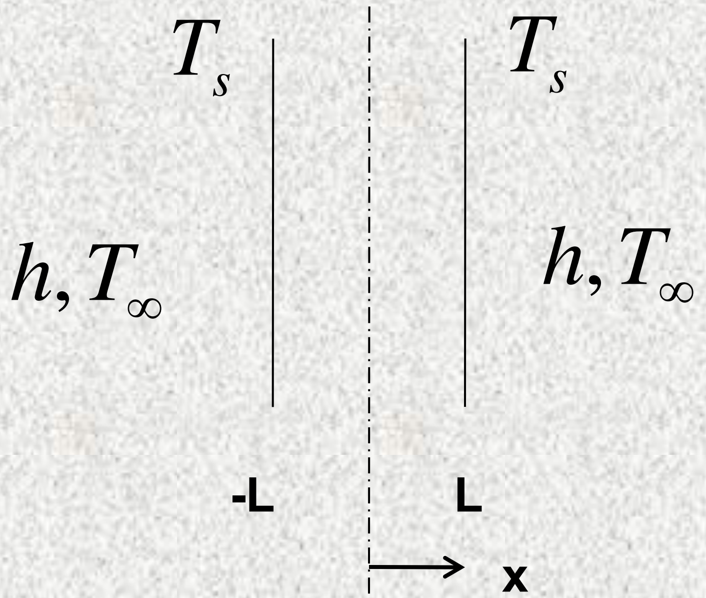
Note:

- Temperature gradient is dependent on x whereas it was independent of x in slab without heat generation

- Temperature distribution in case of heat conduction with heat generation is dependent on thermal conductivity (k). It is independent of k in slab without heat generation.

ONE DIMENSIONAL STEADY STATE CONDUCTION WITH HEAT GENERATION

BC in terms of h same on both sides :



Energy balance gives

$$h(T_L - T_\infty) = \frac{q'''(2L)}{2}$$

Additional BC: $\left.\frac{dT}{dx}\right|_{x=0} = 0$

Governing equation $$\frac{\partial^2 T}{\partial x^2} + \frac{q'''}{k} = 0$$

ONE DIMENSIONAL STEADY STATE CONDUCTION WITH HEAT GENERATION

$$T(x) = \frac{q'''}{k}\frac{L^2}{2}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_2 - T_1}{2}\frac{x}{L} + \frac{T_1 + T_2}{2}$$

$$T(x) = \frac{q'''}{k}\frac{L^2}{2}\left(1 - \frac{x^2}{L^2}\right) + T_s$$

$$\frac{dT}{dx} = \frac{q'''}{k}x$$

$$\left.\frac{dT}{dx}\right|_{x=L} = \frac{q'''L}{k}$$

Boundary Condition:

$$-k\left.\frac{dT}{dx}\right|_{x=L} = h(T_s - T_\infty)$$

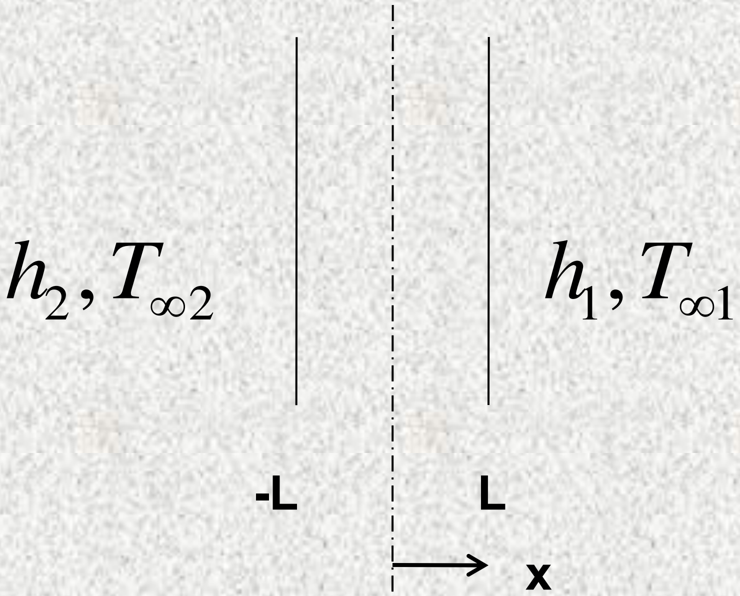
$$T_s = \frac{q'''}{h} + T_\infty$$

Thus

$$T(x) = \frac{q'''}{k}\frac{L^2}{2}\left(1 - \frac{x^2}{L^2}\right) + \frac{q'''L}{h} + T_\infty$$

ONE DIMENSIONAL STEADY STATE CONDUCTION WITH HEAT GENERATION

BC in terms of h known on wall:



$$q^{'''} A \times 2L = h_1 A(T_{w1} - T_{\infty 1}) + h_2 A(T_{w2} - T_{\infty 1})$$

$$\text{Thus,} \quad 2q^{'''} L = h_1 T_{w1} - h_1 T_{\infty 1} + h_2 T_{w2} - h_2 T_{\infty 2}$$

Obtain the solution for this case

THE CYLINDER

Consider a hollow cylinder, whose inner and outer surfaces are exposed to fluids at different temperatures. For steady state conditions with no heat generation, the appropriate form of the heat equation,

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)=0$$

where, for a moment *k* is treated as a variable. The physical significance of this result becomes evident if we also consider the appropriate form of Fourier's law. The rate at which energy is conducted across the cylindrical surface in the solid may be expressed as

$$q_r=-kA\frac{dT}{dr}=-k(2\pi rL)\frac{dT}{dr}$$

Equation A

where $A = 2\pi rL$ is the area normal to the direction of heat transfer.

NOTE: Since, above equation prescribes that the quantity $kr\frac{dT}{dr}$ is independent of *r*, it follows that the conduction heat transfer rate q_r (not the heat flux q_r'') is a constant in the radial direction.

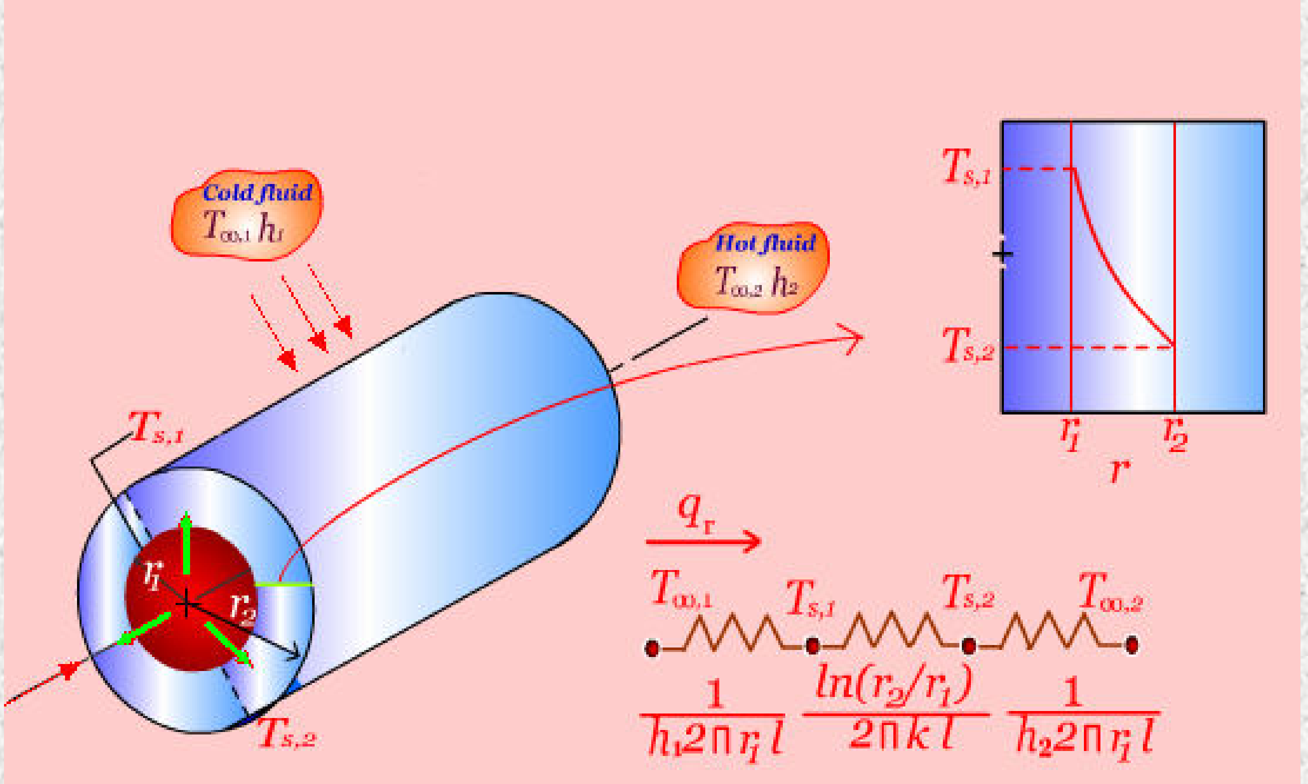


Figure : Hollow Cylinder With Convective Surface Conditions

Assuming the value of *k* to be constant, above equation may be integrated twice to obtain the general solution

$$T(r) = C_1 \ln r + C_2$$

Applying the boundary conditions to the general solution,
i.e. $T(r_1) = T_{s,1}$ and $T(r_2) = T_{s,2}$ we obtain,

$$T_{s,1} = C_1 \ln r_1 + C_2$$

$$T_{s,2} = C_1 \ln r_2 + C_2$$

Solving for C_1 and C_2 and substituting into the general solution, we then obtain

$$T(r) = \frac{T_{s,1} - T_{s,2}}{\ln\left(r_1 / r_2 \right)} \ln\left(\frac{r}{r_2} \right) + T_{s,2}$$

NOTE: that the temperature distribution associated with radial conduction through a cylindrical wall is logarithmic, not linear, as it is for the plane wall. The logarithmic distribution is shown in Figure.

If the temperature distribution equation, is now used with Fourier's law, we obtain the following expression for the heat transfer rate:

$$q_r = \frac{2\pi L k \left(T_{s,1} - T_{s,2} \right)}{\ln\left(r_2 / r_1 \right)}$$

From this result it is evident that, for radial conduction in a cylindrical wall, the thermal resistance is of the form

$$R_{t,cond} = \frac{ln\left(r_2/r_1\right)}{2\pi Lk}$$

This resistance is shown in Figure. Note that since the value of q_r is independent of r , the foregoing result could have been obtained by using the alternative method, that is, by integrating Equation A.

Consider now the composite system. Recalling how we treated the composite plane wall and neglecting contact resistances between the interface, the heat transfer rate may be expressed as

$$q_r = \frac{T_{\infty,1} - T_{\infty,2}}{\left(\frac{1}{2\pi r_1 L h_1}\right) + \left(\frac{ln\left(r_2/r_1\right)}{2\pi k_A L}\right) + \left(\frac{ln\left(r_3/r_2\right)}{2\pi k_B L}\right) + \left(\frac{ln\left(r_4/r_3\right)}{2\pi k_C L}\right) + \left(\frac{1}{2\pi r_4 L h_4}\right)}$$

The above result may also be expressed in terms of an overall heat transfer coefficient. That is,

$$q_r = \frac{T_{\infty,1} - T_{\infty,4}}{R_{tot}} = UA\left(T_{\infty,1} - T_{\infty,4}\right)$$

If *U* is defined in terms of the inside area, *A_1=2πr_1L* Equations 2.47 and 2.48 may be equated to yield

$$U = \frac{1}{\frac{1}{h_1} + \frac{r_1}{k_A} ln\left(r_2 / r_1\right) + \frac{r_1}{k_B} ln\left(r_3 / r_2\right) + \frac{r_1}{k_C} ln\left(r_4 / r_3\right) + \frac{r_1}{r_4}\frac{1}{h_4}}$$

Note:

- ***UA* is constant, while *U* is not**
- **In radial system *q"* is constant, while *q* is not**

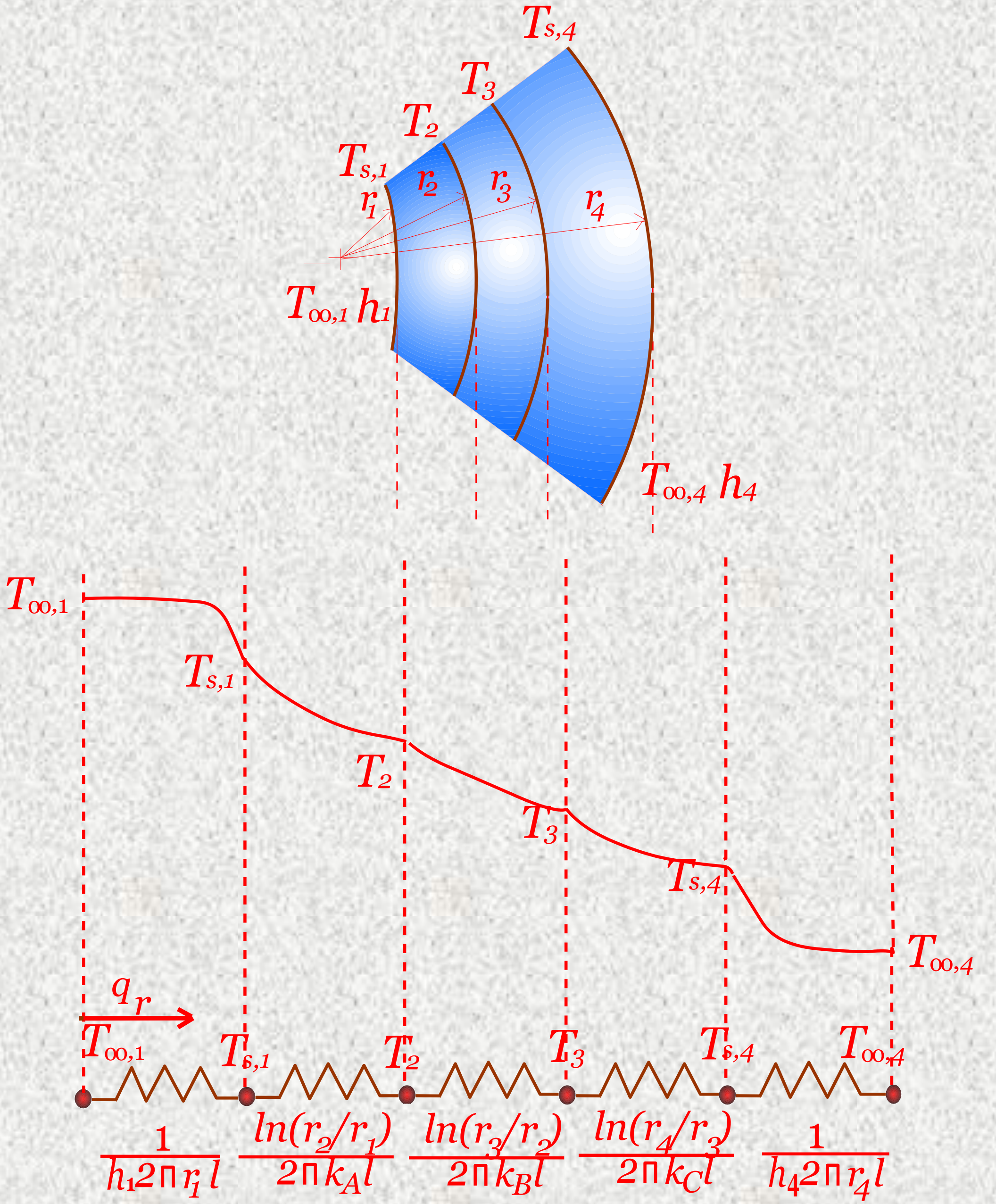


Figure :Temperature Distribution For A Composite Cylindrical Wall

Objectives

- One dimensional steady conduction in sphere is introduced
- The approach is to reduce the heat diffusion equation for the case chosen.
- Using the appropriate boundary conditions, the heat diffusion equation is solved for temperature distribution.
- Concept of critical radius of insulation is presented.

THE SPHERE

Consider a hollow sphere, whose inner and outer surfaces are exposed to fluids at different temperatures (Fig. 2.14).

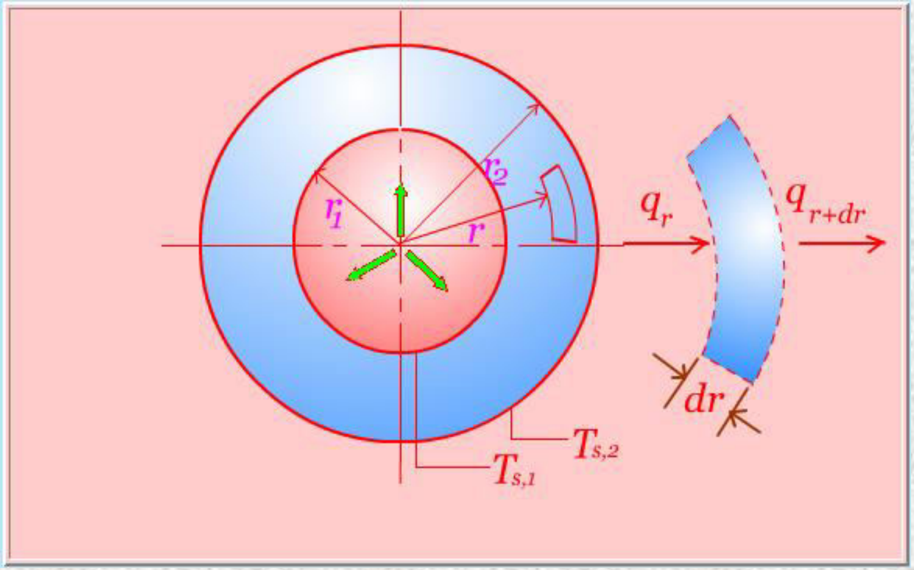


Figure: Conduction in a spherical shell

For steady state conditions with no heat generation, the appropriate form of the heat equation,

$$\frac{1}{r^2}\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right)=0$$

where, for a moment *k* is treated as a variable. The physical significance of this result becomes evident if we also consider the appropriate form of Fourier's law. The rate at which energy is conducted across the cylindrical surface in the solid may be expressed as

$$q_r=-kA\frac{dT}{dr}=-k(4\pi r^2)\frac{dT}{dr}$$

where $A=4\pi r^2$ is the area normal to the direction of heat transfer. Since, above equation 2.50 states that the quantity $kr^2\frac{dT}{dr}$ is independent of *r*, it follows that the conduction heat transfer rate q_r (not the heat flux q_r'') is a constant in the radial direction.

Assuming the value of *k* to be constant, above equation may be integrated twice to obtain the general solution

$$T(r) = \frac{C_1}{r} + C_2$$

Applying, the following boundary conditions

$T(r_1) = T_{s,1}$ and $T(r_2) = T_{s,2}$

we then obtain

$$T_{s,1} = \frac{C_1}{r_1} + C_2$$

$$T_{s,2} = \frac{C_1}{r_2} + C_2$$

Solving for C_1 and C_2 and substituting into the general solution, we then obtain

$$T(r) = T_{s,1} + \frac{T_{s,1} - T_{s,2}}{\left(\frac{1}{r_2} - \frac{1}{r_1}\right)}\left[\frac{1}{r_1} - \frac{1}{r}\right]$$

Note that the temperature distribution associated with radial conduction through a spherical wall is not linear, as it is for the plane wall under the same conditions.

If the temperature distribution equation is now used with Fourier's law, Equation 2.51, we obtain the following expression for the heat transfer rate:

$$q_r = \frac{4\pi\, k\left(T_{s,1} - T_{s,2}\right)}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

From this result it is evident that, for radial conduction in a spherical wall, the thermal resistance is of the form

$$R_{t,cond} = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k}$$

Note that since the value of q_r is independent of r , the above result could have been obtained by using the alternative method, that is, by integration Equation

$$q_r = -kA\frac{dT}{dr} = -k(4\pi r^2)\frac{dT}{dr}$$

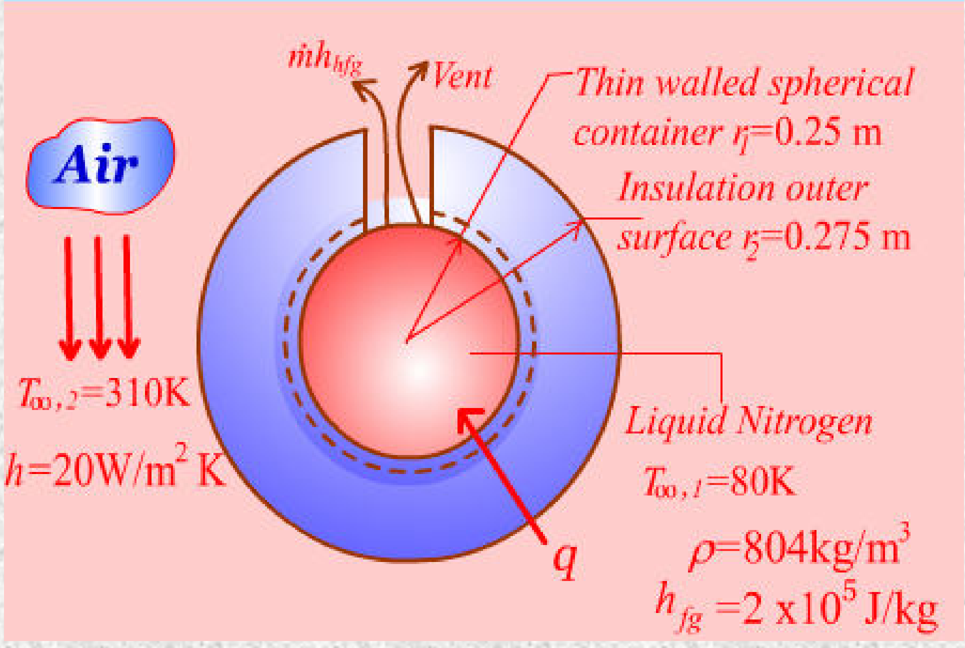
Spherical composites may be treated in much the same way as composite walls and cylinders, where approximate forms of the total resistance and overall heat transfer coefficient may be determined

Problem 2.3

A spherical thin walled metallic container is used to store liquid nitrogen at 80 K. The container has a diameter of 0.5 *m* and is covered with an evacuated, reflective insulation composed of silica powder. The insulation is 25 mm thick, and its outer surface is exposed to ambient air at 310K. The convection coefficient is known to be 20 $W/m^2\,K$. The latent heat of vaporization and the density of the liquid nitrogen are $2 \times 10^5 J/kg$ and 804 kg/m^3, respectively. Thermal conductivity of evacuated silica powder (300 K) is 0.0017 *W/m.K*

- what is the rate of heat transfer to the liquid nitrogen ?
- what is the rate of liquid boil-off ?

Figure



Known: Liquid nitrogen is stored in spherical container that is insulated and exposed to ambient air.

Find:

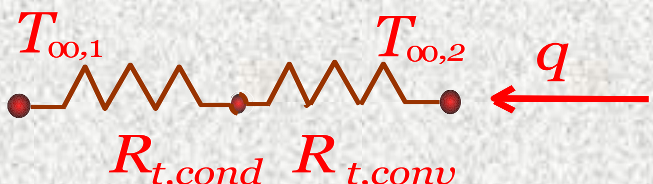
- The rate of heat transfer to the nitrogen.
- The mass rate of nitrogen boil-off.

Assumptions:

1. Steady state conditions
2. One dimensional transfer in the radial direction
3. Negligible resistance to heat transfer through the container wall and from the container to the nitrogen
4. Constant properties
5. Negligible radiation exchange between outer surface of insulation and surroundings

Analysis:

1. The thermal circuit involves a conduction and convection resistance in series and is of the form



where, from Equation 2.55

$$R_{t,cond} = \frac{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}{4\pi k}$$

and from Equation 2.30

$$R_{t,conv} = \frac{1}{hA} = \frac{1}{h\left(4\pi r_2^2\right)}$$

The rate of heat transfer to the liquid nitrogen is then

$$q = \frac{T_{\infty,2} - T_{\infty,1}}{(1/4\pi k)\left[(1/r_1) - (1/r_2)\right] + \left(1/h 4\pi r_2^2\right)}$$

$$q = \frac{310 - 80}{\left(1/4\pi(0.0017)\right)\left[(1/0.25) - (1/0.275)\right] + \left(1/(20)4\pi(0.275)^2\right)}$$

$$q = \frac{230}{17.02 + 0.05} = 13.47\ \mathrm{W}$$

2. Performing an energy balance for a control surface about the nitrogen, it follows from Equation 1.7 that

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

Where $\dot{E}_{in} = q$ and $\dot{E}_{out} = \dot{m} h_{fg}$ is associated with the loss of latent energy due to boiling. Hence,

$$q - m\, h_{fg} = 0$$

and the boil off is,

$$\dot{m} = \frac{q}{h_{fg}} = \frac{13.47}{2\times 10^{5}} = 6.74\times 10^{-5}\ \mathrm{kg/s}$$

The loss per day is

$$\dot{m} = 6.74\times 10^{-5}\ \mathrm{kg/s}\ \times 3600\,\mathrm{s/hours}\times 24\,\mathrm{hours/day}$$

$$\dot{m} = 5.82\,\mathrm{kg/day}$$

or on a volumetric basis

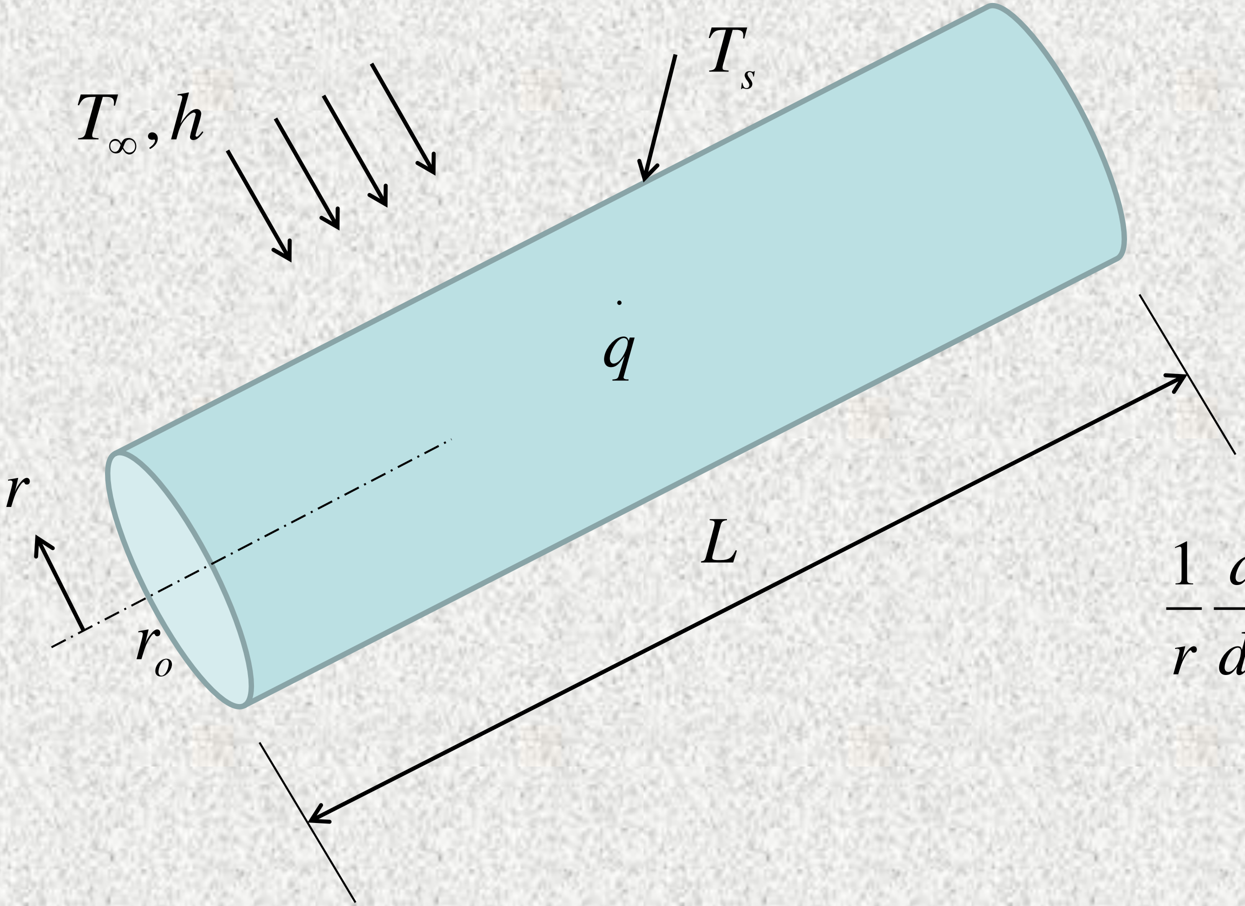
$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{5.82\,\mathrm{kg/day}}{804\,\mathrm{kg/day}} = 0.00724\,\mathrm{m^3/day} = 7.24\,\mathrm{liters/day}$$

Comments:

1. $R_{t,conv} \ll R_{t,cond}$

2. With a container volume of (4/3)(πr_1^3) =0.065 m^3= 65 litres, the daily loss to (7.24liters/65 liters) 100% = 11.14% of capacity.

STEADY STATE HEAT CONDUCTION IN RADIAL SYSTEM WITH HEAT GENERATION



$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k}=0$$

$$\left(r\frac{dT}{dr}\right)=-\frac{\dot{q}}{2k}r^2+C_1$$

$$T(r)=-\frac{\dot{q}}{4k}r^2+C_1\ln r+C_2$$

STEADY STATE HEAT CONDUCTION IN RADIAL SYSTEM WITH HEAT GENERATION

Boundary conditions:

$$\left.\frac{dT}{dr}\right|_{r=0} = 0 \qquad\qquad T(r_o) = T_s$$

$$C_1 = 0 \qquad\qquad C_2 = T_s + \frac{\dot{q}}{4k} r_o^2$$

$$T(r) = \frac{\dot{q}\, r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s$$

STEADY STATE HEAT CONDUCTION IN RADIAL SYSTEM WITH HEAT GENERATION

In terms of outside conditions the BCs may be implemented considering conduction equal to convection:

Thus

$$\dot{q}(\Pi r_o^2 L) = h(2\Pi r_o L)(T_s - T_\infty)$$

$$T_s = T_\infty + \frac{\dot{q} r_o}{2h}$$

$$T(r) = \frac{\dot{q}\, r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right) r^2 + T_\infty + \frac{\dot{q} r_o}{2h}$$

THE CRITICAL RADIUS OF INSULATION

- We know that by adding more insulation to a wall always decreases heat transfer.
- This is expected, since the heat transfer area *A* is constant, and adding insulation will always increase the thermal resistance of the wall without affecting the convection resistance.
- However, adding insulation to a cylindrical piece or a spherical shell, is a different matter.
- The additional insulation increases the conduction resistance of the insulation layer but it also decreases the convection resistance of the surface because of the increase in the outer surface area for convection.
- Therefore, the heat transfer from the pipe may increase or decrease, depending on which effect dominates.

Consider a cylindrical pipe (Figure. 2.15), where,

r_1 **-- outer radius**

T_1 **-- constant outer surface temperature**

k **-- thermal conductivity of the insulation**

r_2 **-- outer radius**

- temperature of surrounding medium

h **- convection heat transfer coefficient**

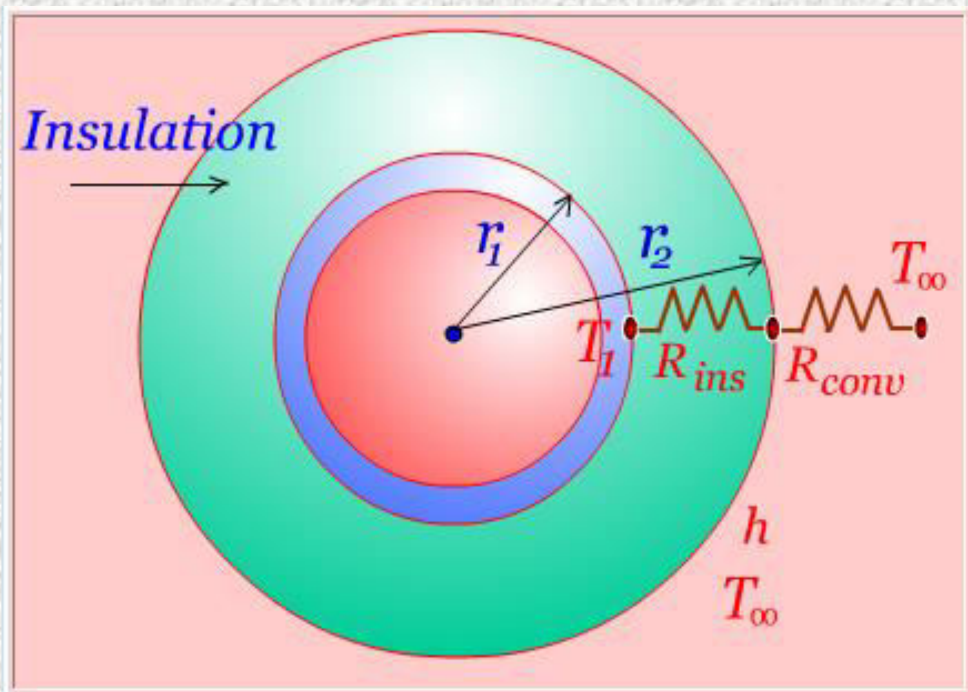


Figure Insulated Cylindrical Pipe

The rate of heat transfer from the insulated pipe to the surrounding air can be expressed as

$$q_r = \frac{(T_1 - T_\infty)}{\frac{ln\left(r_2/r_1\right)}{2\pi Lk} + \frac{1}{h(2\pi r_2 L)}}$$

The variation of heat transfer rate with the outer radius of insulation r_2 is plotted in Figure. The value of r_2 at which heat transfer rate reaches maximum is determined from the requirement that $\frac{dq_r}{dr}$ (zero slope).

Performing the differentiation and solving for r_2 gives us the critical radius of insulation for a cylindrical body to be

$$r_{cr,cylinder} = \frac{k}{h}$$

NOTE: The rate of heat transfer from the cylinder increases with the addition of insulation for $r_2 < r_{cr}$, reaches a maximum when $r_2 = r_{cr}$, and starts to decrease for $r_2 > r_{cr}$. Thus, insulating the pipe may actually increase the rate of heat transfer from the pipe instead of decreasing it when $r_2 < r_{cr}$.

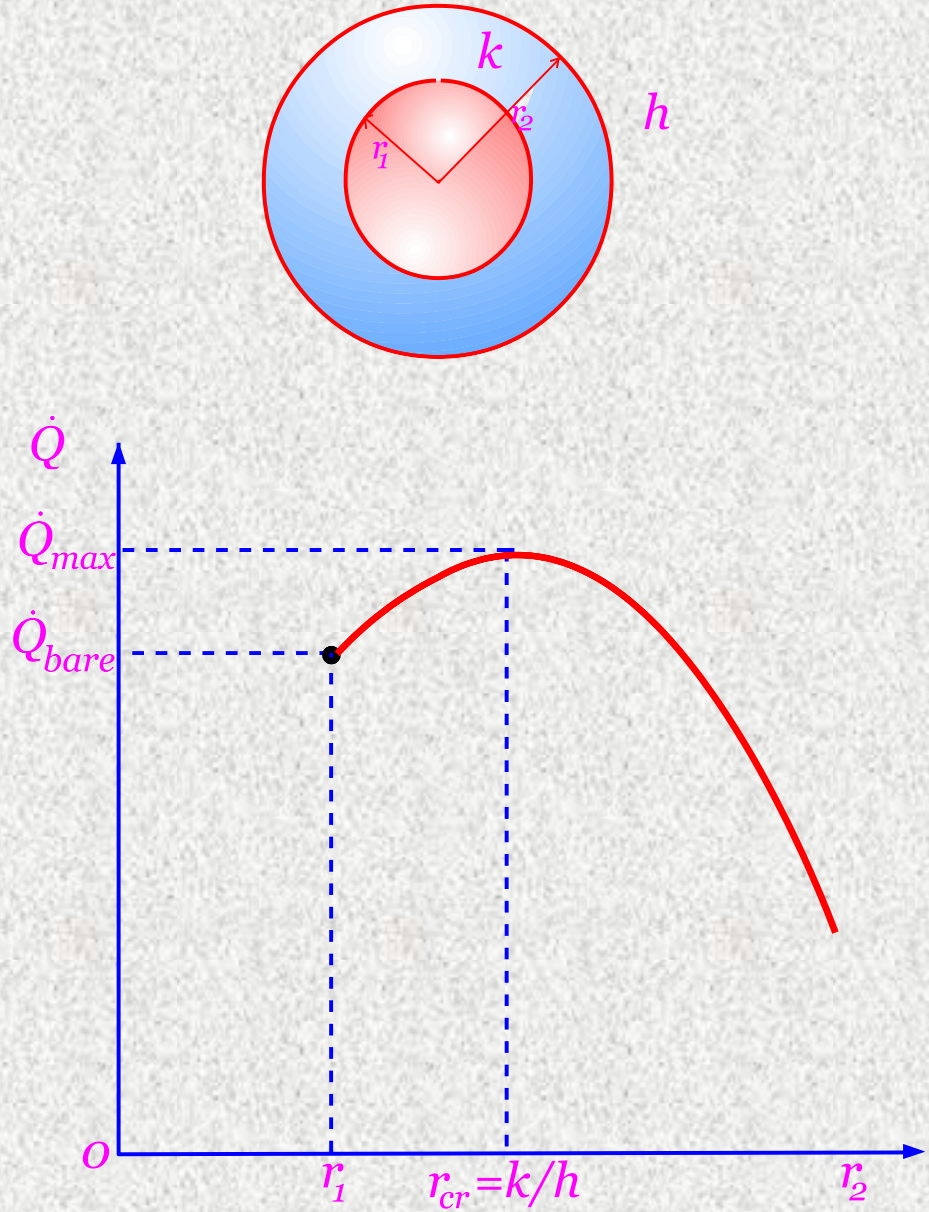


Figure : Variation Of Heat Transfer Rate With Radius

The important question to answer at this point is,

- **Whether we need to be concerned about the critical radius of insulation when insulating hot water pipes or even hot water tanks?**
- **Should we always check and make sure that the outer radius of insulation exceeds the critical radius before we install any insulation?**

  Probably not, as explained below.
- **The value of the critical radius r_{cr} will be the largest when k is large and h is small.**
- **Noting that the lowest value of h encountered in practice is about 5 W/m²K for the case of natural convection of gases**
- **Also, the thermal conductivity of common insulating materials is 0.05 W/m²K,**
- **The largest value of the critical radius we are likely to encounter is**

$$r_{cr} = \frac{k_{max,insulation}}{h_{min}} = \frac{0.05}{5} = 0.01\,m = 10\,mm$$

- **This value would be even smaller when the radiation effects are considered.**

- **The critical radius would be much less in forced convection, often less than 1 *mm*, because of much larger h values associated with forced convection.**
- **Therefore, we can insulate hot water or steam pipes freely without worrying about the possibility of increasing the heat transfer by insulating the pipes.**
- **The radius of electric wires may be smaller than the critical radius.**
- **Therefore, the plastic electrical insulation may actually enhance the heat transfer from electric wires and thus keep their steady operating temperatures at lower and thus safer levels.**

Similarly for a sphere, it can be shown that the critical radius of insulation for a spherical shell is

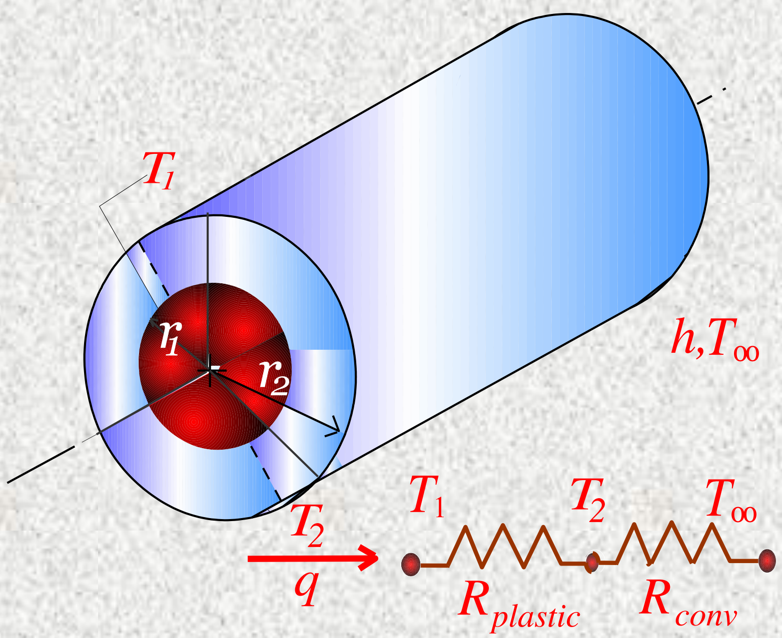
$$r_{cr,sphere} = \frac{2k}{h}$$

where *k* is the thermal conductivity of the insulation and *h* is the convection heat transfer coefficient on the outer surface.

Problem 2.4:

A 3 mm diameter and 6 *m* long electric wire is tightly wrapped with a 2 mm thick plastic cover whose thermal conductivity is $k = 0.15\ W/m.^0C$. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at 27°C with a heat transfer coefficient of $h=12\ W/m^2.^oC$, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.

Figure



Known : Size of the electric wire, thermal conductivity of the wire, current and voltage supplied to the wire, ambient conditions and heat transfer coefficient.

Find :

Convection heat transfer coefficient between the outer surface of the wire and the air in the room.

1. Heat transfer is steady since there is no indication of any change with time.
2. Heat transfer is one dimensional since there is thermal symmetry about the center line and no variation in the axial direction.
3. Thermal conductivities are constant.
4. The thermal contact resistance at the interface is negligible.
5. Heat transfer coefficient incorporates the radiation effects, if any.

Analysis:

Heat is generated in the wire and its temperature rises as a result of resistance heating. We assume heating is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined from

$$\dot{Q} = VI = 8(10) = 80\,\mathrm{W}$$

The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in Schematic. The values of these two resistances are determined to be

$$A_2 = 2\pi r_2 L = 2\pi(0.0035)(6) = 0.132\,\text{m}^2$$

$$R_{\text{conv}} = \frac{1}{hA} = \frac{1}{12(0.132)} = 0.63\,\text{deg C/W}$$

$$R_{\text{plastic}} = \frac{ln(r_2/r_1)}{2\pi kL} = \frac{ln(3.5/1.5)}{2\pi(0.15)6} = 0.15\,\text{deg C/W}$$

and therefore

$R_{total} = R_{total} + R_{conv}$ = 0.63 + 0.15 = 0.78 °C/W

Then the interface temperature can be determined from

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}}$$

$$T_1 = T_\infty + \dot{Q}R_{\text{total}}$$

$$T_1 = 27 + 80(0.78)$$

$$= 89.4°\text{C}$$

Note that we did not involve the electrical wire directly in the thermal resistance network, since the wire involves heat generation.

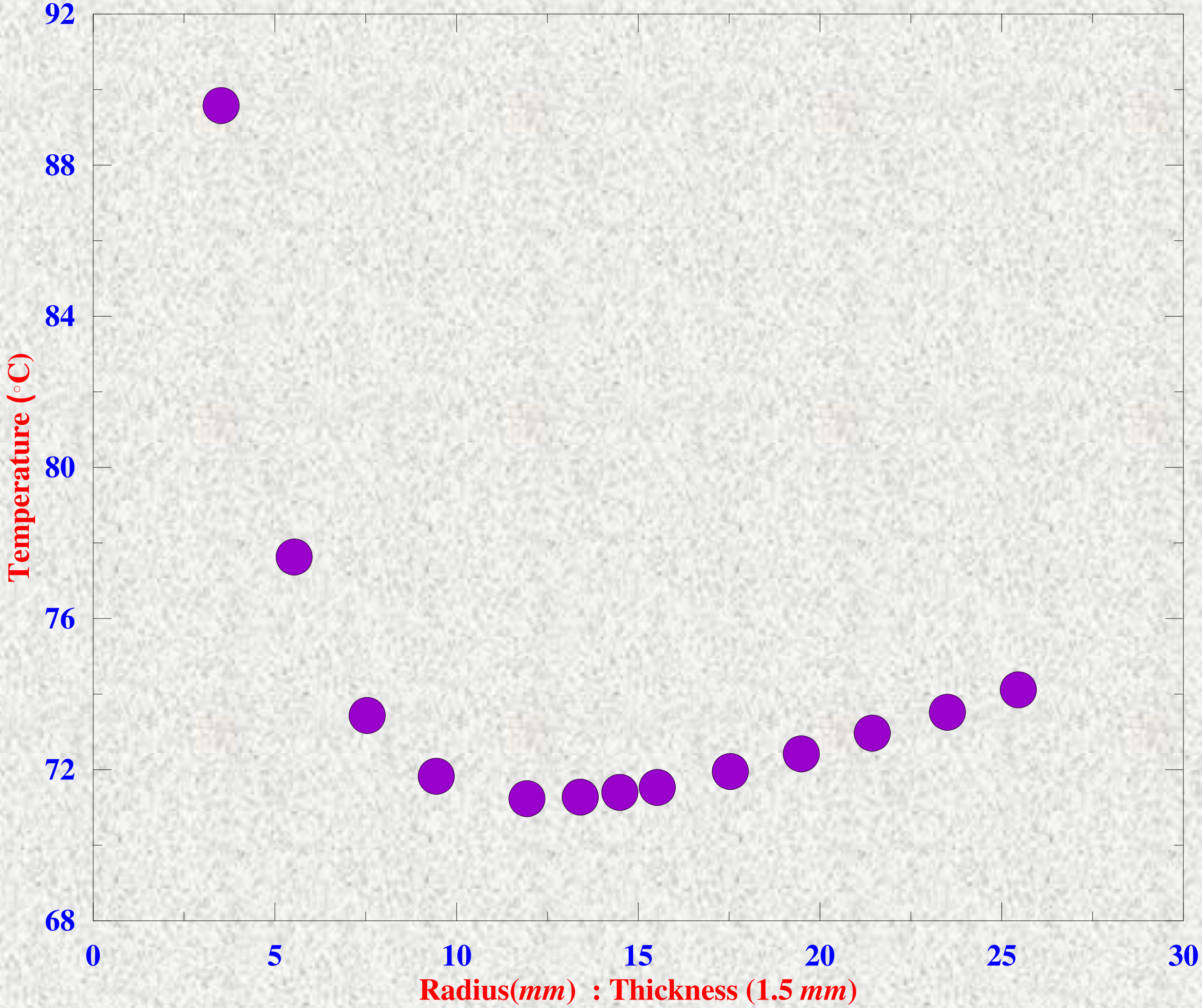
To answer the second part of the question, we need to know the critical radius of insulation of the plastic cover.

$$r_{cr} = \frac{k}{h} = \frac{0.15}{12} = 0.0125\,\text{m} = 12.5\,\text{mm}$$

which is larger than the radius of the plastic cover. Therefore, increasing the thickness of the plastic cover will enhance heat transfer until the outer radius of the cover reaches 12.5 mm. As a result, the rate of heat transfer will increase when the interface temperature T_1 is held constant.

Comments:

It can be shown by repeating the calculations above for a 4 mm plastic cover that the interface temperature drops to 77.54°C when the thickness of the plastic cover is doubled. It can also be shown in a similar manner that the interface reaches a minimum temperature of 71.14°C when the outer radius of the plastic cover equals the critical radius.


92
88
84
80
76
72
68
Temperature (°C)
0
5
10
15
20
25
30
Radius(*mm*) : Thickness (1.5 *mm*)

SUMMARY OF 1-D STEADY STATE CONDUCTION

Many important problems are characterized by one-dimensional steady-state conduction in plane, cylindrical or spherical walls with or without thermal energy generation. Key results for these geometries are summarized in Table on next slide, where ΔT refers to the temperature difference, $T_{s,1}-T_{s,2}$, between the inner and outer surfaces identified in the corresponding figures earlier. In each case, beginning with the heat equation, you should be able to derive the corresponding expression for the temperature distribution, heat flux, heat rate, and thermal resistance.

	Plane Wall	Cylindrical Wall	Spherical Wall
Heat Equation	$\frac{d^2T}{dx^2}=0$	$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)=0$	$\frac{1}{r^2}\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right)=0$
Temperature Distribution	$T_{s,1}-\Delta T\frac{x}{L}$	$\frac{\Delta T}{ln\left(r_1/r_2\right)}ln\left(\frac{r}{r_2}\right)+T_{s,2}$	$T_{s,1}+\frac{\Delta T}{\left(\frac{1}{r_2}-\frac{1}{r_1}\right)}\left[\frac{1}{r_1}-\frac{1}{r}\right]$
Heat Flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\,ln\left(r_2/r_1\right)}$	$\frac{k\,\Delta T}{r^2\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$
Heat Rate (q)	$k\,A\frac{\Delta T}{L}$	$\frac{2\pi\,Lk\,\Delta T}{ln\left(r_2/r_1\right)}$	$\frac{4\pi\,k\,\Delta T}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$
Thermal Resistance ($R_{t,\,cond}$)	$\frac{L}{kA}$	$\frac{ln\left(r_2/r_1\right)}{2\pi\,Lk}$	$\frac{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}{4\pi\,k}$

Table : One-Dimensional Steady State Solutions to the Heat Equation With No Generation

HEAT TRANSFER FROM EXTENDED SURFACES

Objectives

- The significance of enhancing the heat transfer by using fins or extended surfaces is presented initially.
- A general form of the energy equation for one dimensional conditions in an extended surface is derived.

INTRODUCTION

- The term extended surface is commonly used in reference to a solid that experiences energy transfer by conduction within its boundaries, as well as energy transfer by convection (and/or radiation) between its boundaries and the surroundings. (Fig. 3.1)
- A strut is used to provide mechanical support to two walls that are at different temperatures.
- A temperature gradient in the *x*-direction sustains heat transfer by conduction internally, at the same time there is energy transfer by convection from the surface.

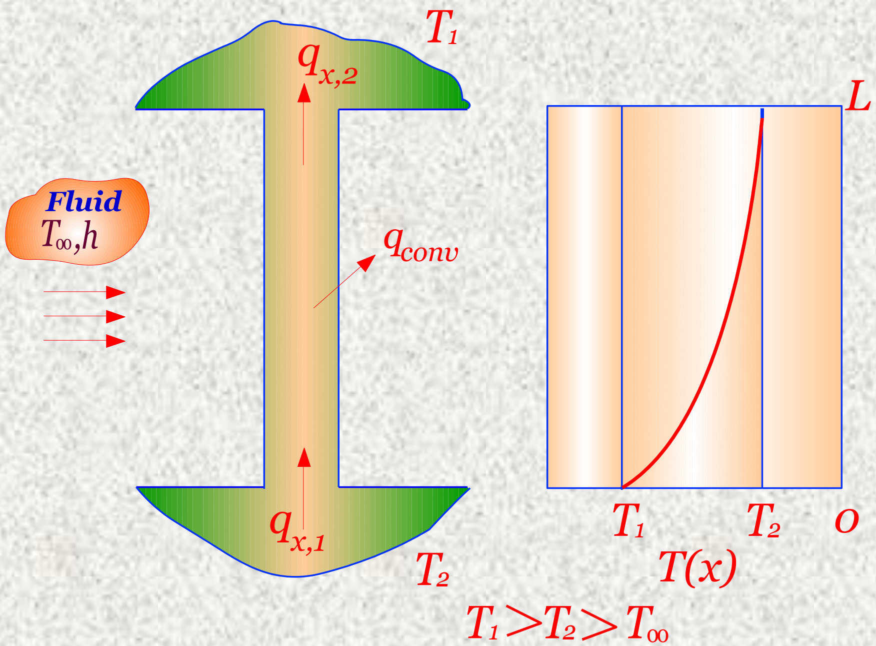


Figure: Combined Conduction And Convection In A Structural Element

- **The most frequent application is one in which an extended surface is used specifically to enhance the heat transfer rate between a solid and an adjoining fluid.**

- **Such an extended surface is termed a fin.**

Consider a plane wall of Figure (next slide).

where,

T_s -- the surface temperature

T_∞ -- temperature of surrounding medium

The rate of heat transfer is given by Newton's law of cooling as

$$Q_{conv} = h A (T_s - T_\infty)$$

where

A -- the heat transfer area

h -- is the convection heat transfer coefficient.

NOTE:

If T_s is fixed, there are two ways in which the heat transfer rate may be increased.

The convection coefficient *h* could be increased by

- **Increasing the fluid velocity**
- **The fluid temperature T_∞ could be reduced**

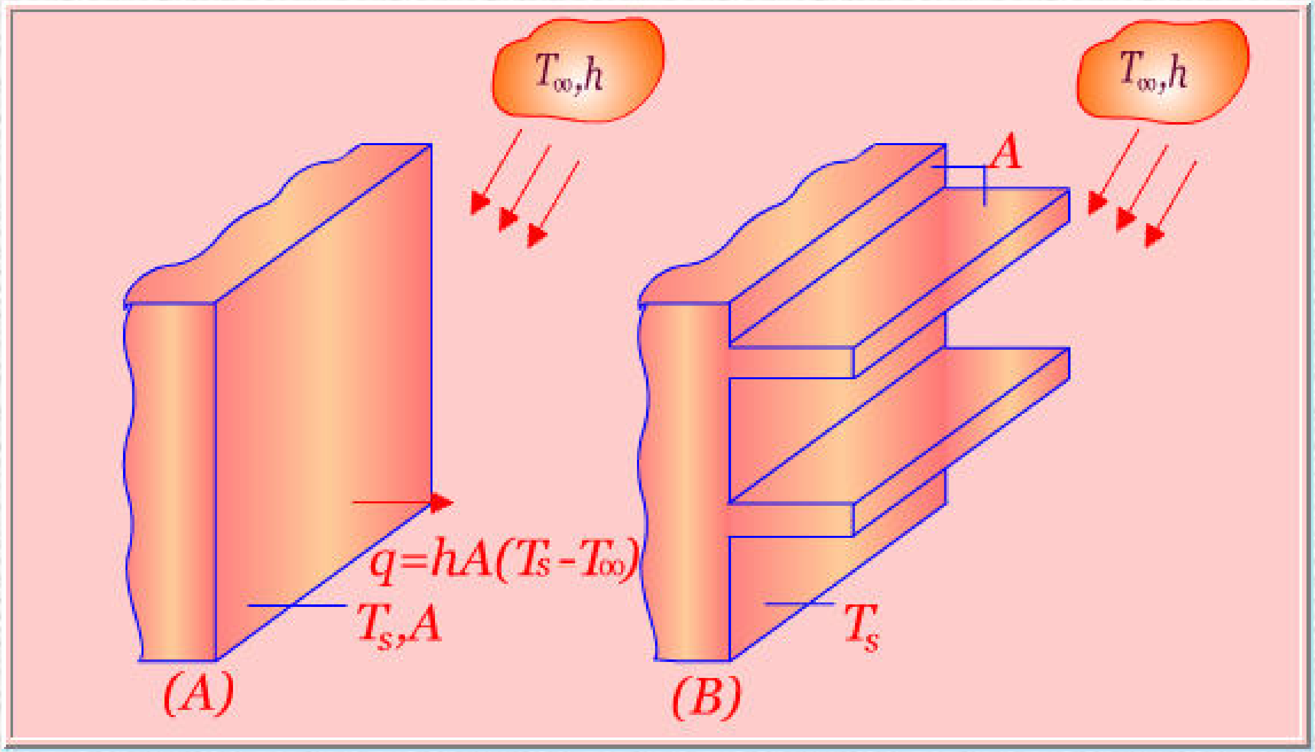


Figure: Use Of Fins To Enhance Heat Transfer From A Plane Wall (A) Bare Surface (B) Finned Surface

LIMITATIONS:

- **Many situations would be encountered in which increasing *h* to the maximum possible value is either insufficient to obtain the desired heat transfer rate or the associated costs are prohibitively high.**

- **Such costs are comprised of the blower or pump power requirements needed to increase *h* through increased fluid motion.**

- **Moreover, the second option of reducing T_∞ is often impractical.**

A third option.

- **That is, the heat transfer rate may be increased by increasing the surface area across which the convection occurs.**

- **This may be done by providing fins that extend from the wall into the surrounding fluid.**

- **The thermal conductivity of the fin material has a strong effect on the temperature distribution along the fin and therefore influences the degree to which the heat transfer rate is enhanced.**

- **Ideally, the fin material should have a large thermal conductivity to minimize temperature variations from its base to its tip.**

- **In the limit of infinite thermal conductivity, the entire fin would be at the temperature of the base surface, thereby providing the maximum possible heat transfer enhancement.**

APPLICATIONS:

There are several fin applications,

- the arrangement for cooling engine heads on motorcycles and lawn-mowers or
- for cooling electric power transformers
- the tubes with attached fins used to promote heat exchange between air and the working fluid of an air conditioner.

Two common finned tube arrangements are shown in Figure (next slide)

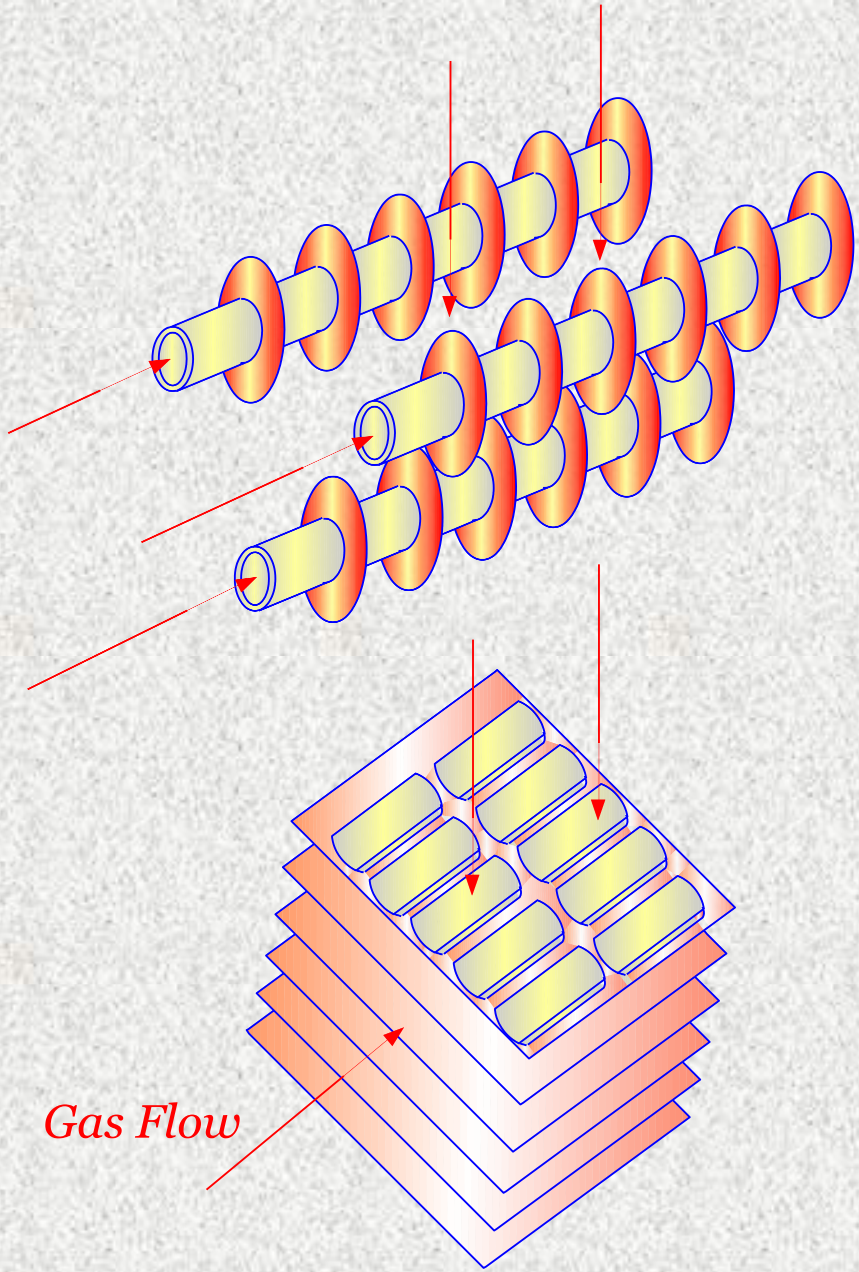


Figure: Schematic Of Typical Finned Tube Heat Exchangers

Different fin configurations are:

- **A straight fin is any extended surface that is attached to a plane wall. It may be of uniform cross sectional area, or its cross sectional area may vary with the distance *x* from the wall.**

- **An annular fin is one that is circumferentially attached to a cylinder, and its cross section varies with radius from the centerline of the cylinder.**

- **The foregoing fin types have rectangular cross sections, whose area may be expressed as a product of the fin thickness *t* and the width *w* for straight fins or the circumference for annular fins.**

- **In contrast a pin fin, or spine, is an extended surface of circular cross section.**

- **Pin fins may also be of uniform or non-uniform cross section.**

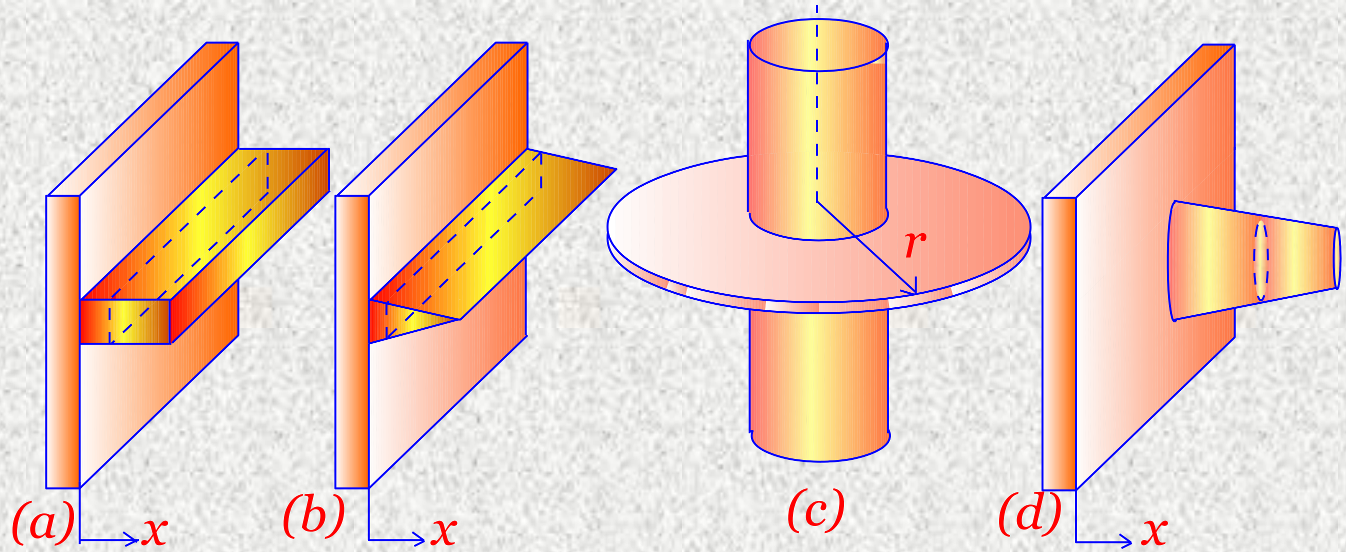


Figure: Fin Configurations (A) Straight Fin Of Uniform Cross Section (B) Straight Fin Of Non-Uniform Cross Section (C) Annular Fin (D) Pin Fin

In any application, selection of a particular fin configuration may depend on

- space,
- weight,
- manufacturing and
- cost considerations,
- the extent to which the fins reduce the surface convection coefficient and
- increase in the pressure drop associated with flow over the fins.

A GENERAL CONDUCTION ANALYSIS

To determine the heat transfer rate associated with a fin, we must first obtain the temperature distribution along the fin. We begin our analysis by performing an energy balance on an appropriate differential element. Consider the extended surface of Figure. The analysis is simplified if certain assumptions are made.

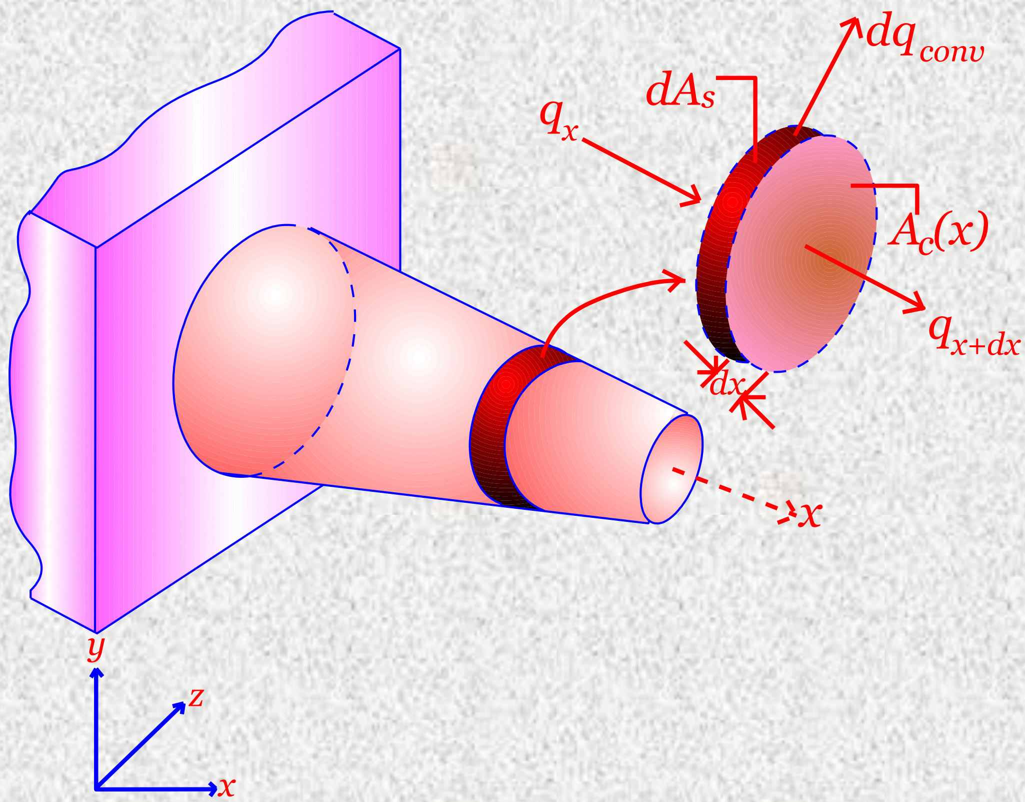


Figure: Energy Balance for an Extended Surface

Assumptions

- Heat transfer is assumed to be in only one dimensional i.e., in the longitudinal (x) direction, even though conduction within the fin is actually two dimensional.
- The rate at which the energy is convected to the fluid from any point on the fin surface must be balanced by the rate at which the energy reaches that point due to conduction in the transverse (y,z) direction. However, in practice the fin is thin and temperature changes in the longitudinal direction are much larger than those in the transverse direction.
- Steady state conditions are assumed.
- Thermal conductivity is assumed to be constant .
- Radiation from the surface is assumed to be negligible .
- Convection heat transfer coefficient is assumed to be uniform over the surface.

Applying the conservation of energy requirement to the differential element, we obtain

$$q_x = q_{x+dx} + dq_{conv}$$

From Fourier's law we know that

$$q_x = -kA_c \frac{dT}{dx}$$

Where A_c is the cross-sectional area, which may vary with x . Since the conduction heat rate at $x+dx$ may be expressed as

$$q_{x+dx} = q_x + \frac{dq_x}{dx} dx$$

It follows that

$$q_{x+dx} = -kA_c \frac{dT}{dx} - k \frac{d}{dx}\left(A_c \frac{dT}{dx} \right) dx$$

The convection heat transfer rate may be expressed as

$$dq_{conv} = h\,dA_s\left(T - T_\infty\right)$$

Where dA_s is the surface area of the differential element.

Substituting the foregoing rate equations into the energy balance equation , we obtain

$$\frac{d}{dx}\left(A_c\frac{dT}{dx}\right) - \frac{h}{k}\frac{dA_s}{dx}\left(T - T_\infty\right) = 0$$

or

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)\left(T - T_\infty\right) = 0$$

This result provides a general from of the energy equation for one dimensional condition in an extended surface. Its solution for appropriate boundary conditions would provide the temperature distribution, which could then be used with Equation of Fourier's law of heat conduction to calculate the conduction rate at any distance *x*.

Objectives

- The temperature distribution for rectangular fin and pin fin with various boundary conditions is obtained from the general form of the energy equation for an extended surface which we derived.

FINS OF UNIFORM CROSS SECTIONAL AREA

Consider the simplest case of straight rectangular and pin fins of uniform cross section (Figures below). Each fin is attached to a base surface of temperature $T(0)=T_b$ and extends into a fluid of temperature T_∞.

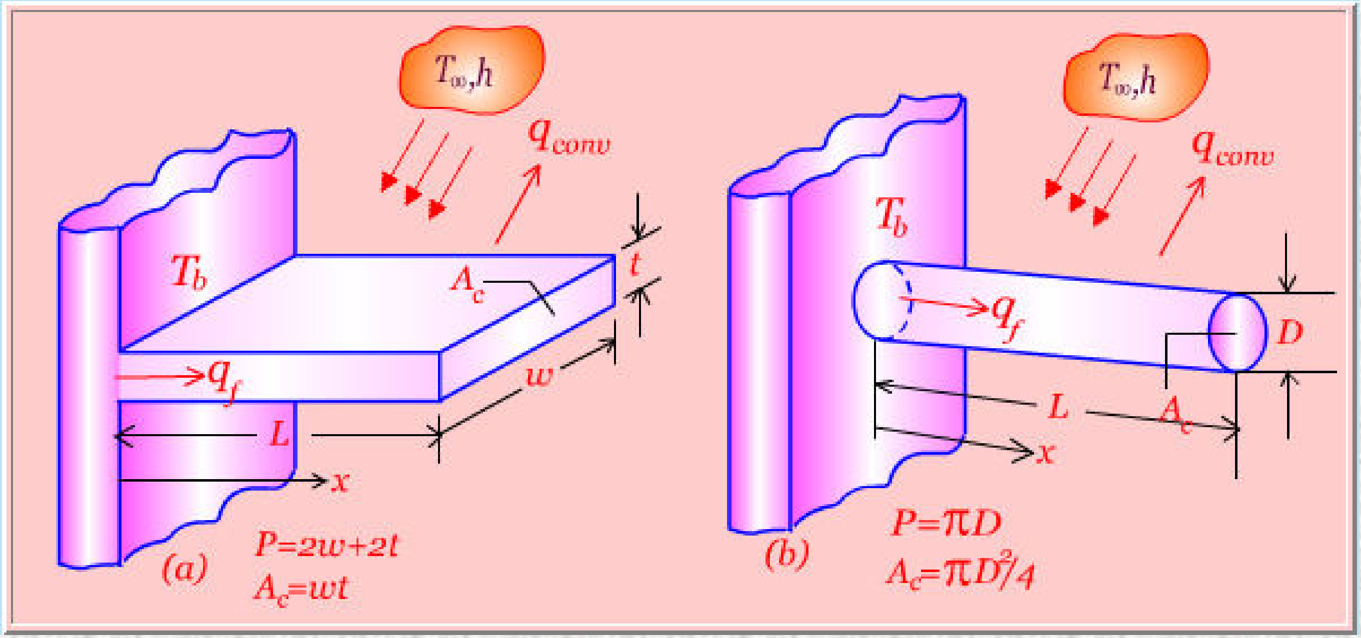


Figure: Fins of Uniform Cross Section (a) Rectangular Fin (b) Pin Fin

For the prescribed fins,

- A_c is a constant and $A_s = Px$,
- A_s is the surface area measured from the base to x and
- P is the fin perimeter.

Accordingly, with $\frac{dA_c}{dx} = 0$ and $\frac{dA_s}{dx} = P$, Governing Equation reduces to

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

To simplify this equation, we transform the dependent variable by defining the excess temperature θ as

$$\theta(x) = T(x) - T_\infty$$

where, since T_∞ is a constant, $\frac{d\theta}{dx} = \frac{dT}{dx}$. Substituting

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$ where $$m^2 = \frac{hP}{kA_c}$$

Above equation is linear, homogenous, second order differential equation with constant coefficients. Its general solution is of the form

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

To evaluate the constant C_1 and C_2 of this equation , it is necessary to specify appropriate boundary conditions. One such condition may be specified in terms of the temperature at the base of the fin (*x=0)*

$$\theta(0) = T_b - T_\infty = \theta_b$$

The second condition, specified at the fin tip (*x=L*), may correspond to any one of the four different physical conditions.

- Convection heat transfer from the fin tip
- Adiabatic condition at the fin tip
- Prescribed temperature maintained at the fin tip
- Infinite fin (very long fin)

Case B, Adiabatic condition at the fin tip

The assumption that the convective heat loss from the fin tip is negligible reduces to the condition that the tip may be treated as adiabatic and we obtain

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

$$\theta = C_1 e^{mx} + C_2 e^{-mx}$$

$$\theta_b = C_1 + C_2 \quad at \quad x = 0$$

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0 \qquad mC_1 e^{mL} - mC_2 e^{-mL} = 0;\ C_1 e^{mL} - C_2 e^{-mL} = 0$$

$$C_2 = C_1 e^{2mL}$$

$$\theta_b = C_1 + C_1 e^{2mL} \Rightarrow C_1 = \frac{\theta_b}{1 + e^{2mL}}$$

$$\frac{\theta}{\theta_b} = \frac{e^{mx}}{1+e^{2mL}} + \frac{e^{2mL}e^{-mx}}{1+e^{2mL}} \Rightarrow \frac{\theta}{\theta_b} = \frac{e^{mx}}{1+e^{2mL}} + \frac{e^{-mx}}{\frac{1}{e^{2mL}}+1}$$

$$\frac{\theta}{\theta_b} = \frac{e^{mx}}{1+e^{2mL}} + \frac{e^{-mx}}{1+e^{-2mL}}; \qquad \frac{\theta}{\theta_b} = \frac{e^{mx}.e^{-mL}}{e^{-mL}+e^{mL}} + \frac{e^{-mx}.e^{mL}}{e^{mL}+e^{-mL}};$$

$$\frac{\theta}{\theta_b} = \frac{e^{m(x-L)}}{e^{-mL}+e^{mL}} + \frac{e^{-m(x-L)}}{e^{mL}+e^{-mL}}$$

$$\frac{\theta}{\theta_b} = \frac{e^{m(x-L)}+e^{-m(x-L)}}{e^{-mL}+e^{mL}} \qquad \cosh x = \frac{e^{x}+e^{-x}}{2}$$

$$\frac{\theta}{\theta_b} = \frac{\cosh[m(x-L)]}{\cosh mL}$$

$$Q = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = -kA_c \left.\frac{d\theta}{dx}\right|_{x=0} \qquad \frac{\theta}{\theta_b} = \frac{e^{mx}}{1+e^{2mL}} + \frac{e^{-mx}}{1+e^{-2mL}}$$

$$\left.\frac{d\theta}{dx}\right|_{x=0} = \theta_b\left[\frac{m}{1+e^{2mL}} - \frac{m}{1+e^{-2mL}}\right]$$

$$\left.\frac{d\theta}{dx}\right|_{x=0} = m\,\theta_b\left[\frac{1}{1+e^{2mL}} - \frac{1}{1+e^{-2mL}}\right]$$

$$Q = -kA_c \left.\frac{d\theta}{dx}\right|_{x=0} = -kA_c m\,\theta_b\left[\frac{1}{1+e^{2mL}} - \frac{1}{1+e^{-2mL}}\right] \qquad m = \sqrt{\frac{hP}{kA}}$$

$$Q = -kA_c\sqrt{\frac{hP}{kA_c}}\theta_b\left[\frac{e^{-mL}}{e^{-mL}+e^{mL}} - \frac{e^{mL}}{e^{mL}+e^{-mL}}\right]$$

$$Q = -kA_c\sqrt{\frac{hP}{kA_c}}\theta_b\left[\frac{e^{-mL}}{e^{-mL}+e^{mL}} - \frac{e^{mL}}{e^{mL}+e^{-mL}}\right]$$

$$Q = \sqrt{hPkA_c}\,\theta_b\left[\frac{e^{mL}-e^{-mL}}{e^{-mL}+e^{mL}}\right]$$

$$Q = \sqrt{hPkA_c}\,\theta_b Tanh mL$$

Case A, Convection Heat Transfer From the Fin Tip

Applying an energy balance to a control surface about this tip, we obtain

$$hA_c\left[T(L)-T_\infty\right] = -kA_c\left.\frac{dT}{dx}\right|_{x=L}$$

$$h\theta(L) = -k\left.\frac{d\theta}{dx}\right|_{x=L}$$

That is, the rate at which energy is transferred to the fluid by convection from the tip must equal the rate at which energy reaches the tip by conduction through the fin.

Thus,

$$\theta_b = C_1 + C_2$$

and

$$h(C_1 e^{mL} + C_2 e^{-mL}) = km(C_2 e^{-mL} - C_1 e^{mL})$$

Solving for C_1 and C_2, it may be shown, after some manipulation, that

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$$

The form of this temperature distribution is shown schematically in Figure below. Note that the magnitude of the temperature gradient decreases with increasing x. This trend is an effect of the reduction in the conduction heat transfer $q_x(x)$ with increasing x due to continuous convection loss from the fin surface.

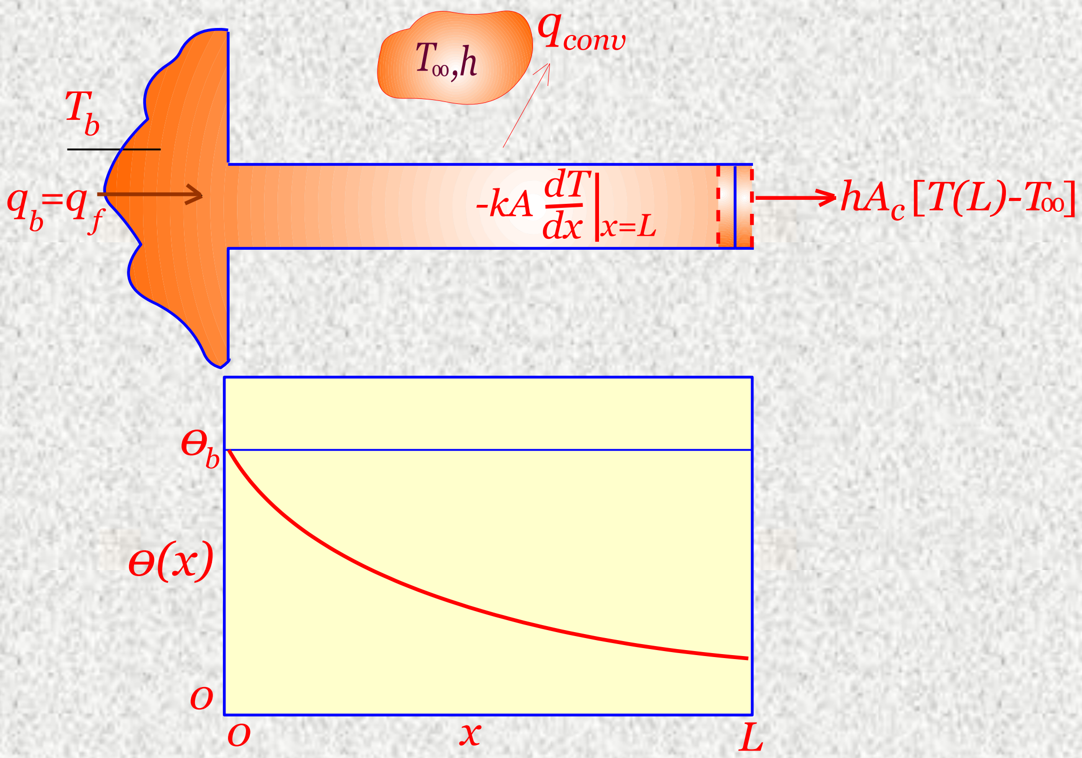


Figure: Conduction And Convection In A Fin Of Uniform Cross Section

Also, we need to find the total heat transferred by the fin. From Figure it is evident that the fin heat transfer rate q_f may be evaluated by applying Fourier's law at the fin base. That is,

$$q_f = q_b = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = -k\,A_c \left.\frac{d\theta}{dx}\right|_{x=0}$$

Hence, knowing the temperature distribution, *q(x)*, q_f may be evaluated, giving

$$q_f = \sqrt{h\,P\,k\,A_c}\,\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

Case B, Adiabatic condition at the fin tip

The assumption that the convective heat loss from the fin tip is negligible reduces to the condition that the tip may be treated as adiabatic and we obtain

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

Substituting and dividing by *m* , we then obtain

$$C_1 e^{mL} - C_2 e^{-mL} = 0$$

Solving for C_1 and C_2 and substituting the results , we obtain

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

Using this temperature distribution , the fin heat transfer rate is

$$q_f = \sqrt{hPkA_c}\,\theta_b \,Tanh\, mL$$

Case C, Prescribed temperature maintained at the fin tip

The assumption that the fin tip is maintained at a prescribed temperature reduces to the following boundary condition

$$\theta(L) = \theta_L$$

Substituting from Equation 3.13, we then obtain

$$\theta_L = C_1 e^{mL} + C_2 e^{-mL} \tag{3.27}$$

Using this expression with Equation 3.17 to solve for C_1 and C_2 and substituting the results into Equation 3.13, we obtain

$$\frac{\theta}{\theta_b} = \frac{\left(\theta_L / \theta_b\right) \sinh mx + \sinh m(L - x)}{\sinh mL} \tag{3.28}$$

Using this temperature distribution with Equation 3.20, the fin heat transfer rate is

$$q_f = \sqrt{h P k A_c}\, \theta_b \frac{\cosh mL - \left(\theta_L / \theta_b\right)}{\sinh mL} \tag{3.29}$$

Case D, Infinite fin (very long fin)

The very long fin situation is an interesting extension of the fin tip maintained at a prescribed temperature.

As $L \to \infty, \theta_L \to 0$, the Equation 3.28 reduces to

$$\frac{\theta}{\theta_b} = \frac{\sinh m(L-x)}{\sinh mL} \qquad (3.30)$$

The above equation reduces to

$$\frac{\theta}{\theta_b} = \frac{\sinh m(L-x)}{\sinh mL} = \frac{e^{m(L-x)} - e^{-m(L-x)}}{e^{mL} - e^{-mL}} = \frac{e^{mL}.e^{-mx} - e^{-mL}.e^{mx}}{e^{mL} - e^{-mL}} \qquad (3.31)$$

As $L \to \infty, e^{-mL} \to 0$, the Equation 3.31 reduces to

$$\frac{\theta}{\theta_b} == \frac{e^{mL}.e^{-mx}}{e^{mL}} = e^{-mx} \qquad (3.32)$$

The fin heat transfer rate given by Equation 3.29 reduces to

$$q_f = \sqrt{hPkA_c}\,\frac{\cosh mL}{\sinh mL} = \sqrt{hPkA_c}\,\frac{e^{mL} + e^{-mL}}{e^{mL} - e^{-mL}} \qquad (3.33)$$

As, $L \to \infty, e^{-mL} \to 0$ the Equation 3.33 reduces to

$$q_f = \sqrt{hPkA_c}\,\theta_b \frac{\cosh mL}{\sinh mL} = \sqrt{hPkA_c}\,\theta_b \frac{e^{mL}}{e^{mL}} = \theta_b \sqrt{hPkA_c} \qquad (3.34)$$

The foregoing results are summarized in Table 3.1.

Table 3.1 Temperature Distribution and heat

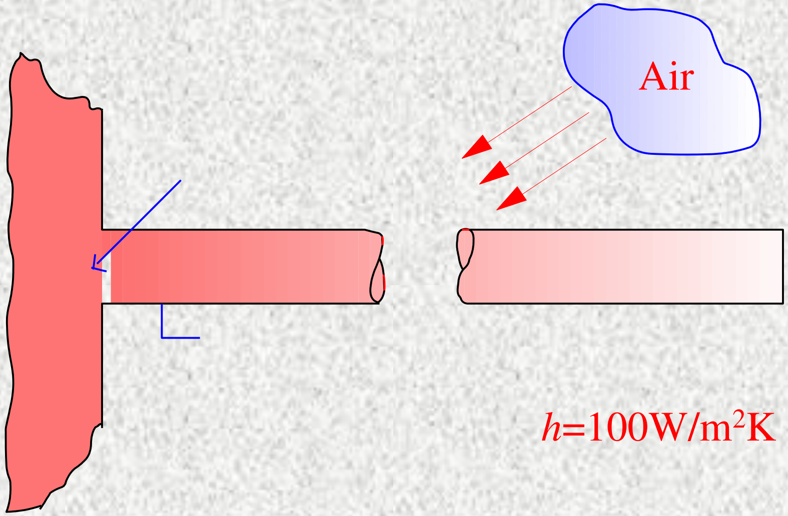
Case	Tip condition	Temperature Distribution $\frac{\theta}{\theta_b}$	Fin Heat Transfer Rate q_f
A	Convection heat transfer $h A_c \left[T(L) - T_\infty\right] = -kA_c \left.\frac{dT}{dx}\right\|_{x=L}$	$\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$\theta_b \sqrt{hPkA_c}\,\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic $\left.\frac{d\theta}{dx}\right\|_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$\sqrt{hPkA_c}\,\theta_b \tanh mL$
C	Prescribed temperature $\theta(L) = \theta_L$	$\frac{\left(\theta_L/\theta_b\right)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$\sqrt{hPkA_c}\,\theta_b \frac{\cosh mL - \left(\theta_L/\theta_b\right)}{\sinh mL}$
D	Infinite fin ($L \to \infty$)	e^{-mx}	$\theta_b \sqrt{hPkA_c}$

Problem 3.1:

A very long rod 5 mm in diameter has one end maintained at 100°C. The surface of the rod is exposed to ambient air at 25°C with a convection heat transfer coefficient of 100 W/m²K.

- Determine the temperature distributions along rods constructed from pure copper, 2024 aluminium alloy and type AISI 316 stainless steel. What are the corresponding heat losses from the rods?
- Estimate how long the rods must be for the assumption of infinite length to yield an accurate estimate of the heat loss

Figure:



Known : A long circular rod exposed to ambient air.

Find :

- Temperature distribution and heat loss when rod is fabricated from copper, an aluminum alloy, or stainless steel.
- How long rods must be to assume infinite length.

Assumptions:

- Steady state conditions
- One dimensional conduction along the rod
- Constant properties
- Negligible radiation exchange with surroundings
- Uniform heat transfer coefficient

Properties : At $[T = (T_b + T_\infty)/2 = 62.5^oC = 335\ K]$ Copper: k = 398 W/m.K; Aluminium: k = 180 W/m.K; Stainless steel, k = 14 W/m.K

Analysis:

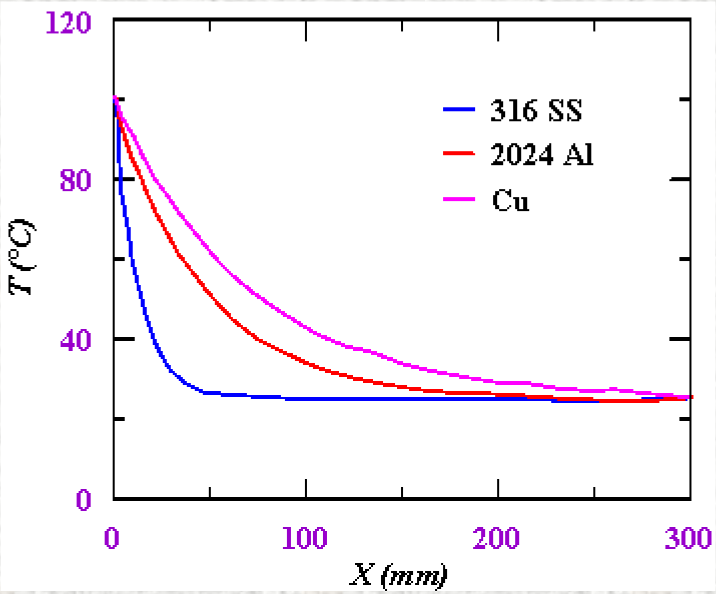
1. Subject to the assumption of an infinitely long fin, the temperature distributions are determined from equation 3.32, which may be expressed as,

$$T = T_\infty + (T_b - T_\infty)e^{-mx}$$

where

$$m = \left(\frac{hP}{kA_c}\right)^{1/2} = \left(\frac{4h}{kD}\right)^{1/2}$$

Substituting for *h* and *D* , as well as for the thermal conductivities of copper, the aluminium alloy and the stainless steel, respectively, the values of *m* are 14.2, 21.2 and 75.6 m^{-1}. The temperature distributions may then be computed and plotted as follows.



From these distributions, it is evident that there is little additional heat transfer associated with extending the length of the rod much beyond 50, 200 and 300 mm, respectively, for the stainless steel, the aluminium alloy and the copper.

From Equation 3.34 the heat loss is,

$$q_f = \theta_b \sqrt{h\,P\,k\,A_c} = \left[100 \times \pi \times 0.005 \times 398 \times \frac{\pi}{4}(0.005)^2\right]^{1/2}(100-25)$$

$$q_f = 8.3\,\mathrm{W}$$

Similarly, for the aluminium alloy and stainless steel, respectively, the heat rates are q_f = 5.6 W and 1.6 W

2. Since there is no heat loss from the tip of an infinitely long rod, an estimate of the validity of the approximation may be made by comparing Equations 3.25 and 3.34. to a satisfactory approximation, the expressions provide equivalent results if *TanhmL* $\geq$ 2.65. Hence a rod may be assumed to be infinitely long if

$$L \geq L_\infty = \frac{2.65}{m} = 2.65\left(\frac{k\,A_c}{h\,P}\right)^{1/2}$$

For copper,

$$L_\infty = 2.65\left(\frac{398\times(\pi/4)\times(0.005)^2}{100\times\pi\times0.005}\right)^{1/2} = 0.19\,\text{m}$$

Results for the aluminium alloy and stainless steel are L_∞ = 0.13 m and L_∞ = 0.04 m, respectively.

Comments:

The above results suggest that the fin heat transfer rate may accurately be predicted from the infinite fin approximation if $mL \geq$ 2.65

However, if the infinite fin approximation is to accurately predict the temperature distribution *T(x)* , a larger value of *mL* would be required. This value may be inferred from Equation 3.32 and the requirement that the tip temperature be very close to the fluid temperature. Hence, if we require that , it follows that *mL* > 4.6, in which case $\theta(L)/\theta_b = exp(-mL) < 0.01$ L_∞ 0.33,0.23 and 0.07 m for the copper aluminium alloy, and stainless steel, respectively.

Objectives

- The concept of fin efficiency, effectiveness is introduced in order to compare various fin configurations.
- Also, the proper length of fin from practical point of view is presented.

FIN EFFICIENCY

Consider the surface of the plane wall

- at temperature T_b
- exposed to a medium at temperature T_∞.
- Heat is lost from the surface to the surrounding medium by convection with a heat transfer coefficient of *h*.

Neglecting radiation, heat transfer from a surface area *A* is expressed as

$Q_{conv} = h\,A\,(T_s - T_\infty)$. Now let us consider a fin of constant cross sectional area $A_c = A_b$ and length *L* that is attached to the surface with a perfect contact (Figure on next slide).

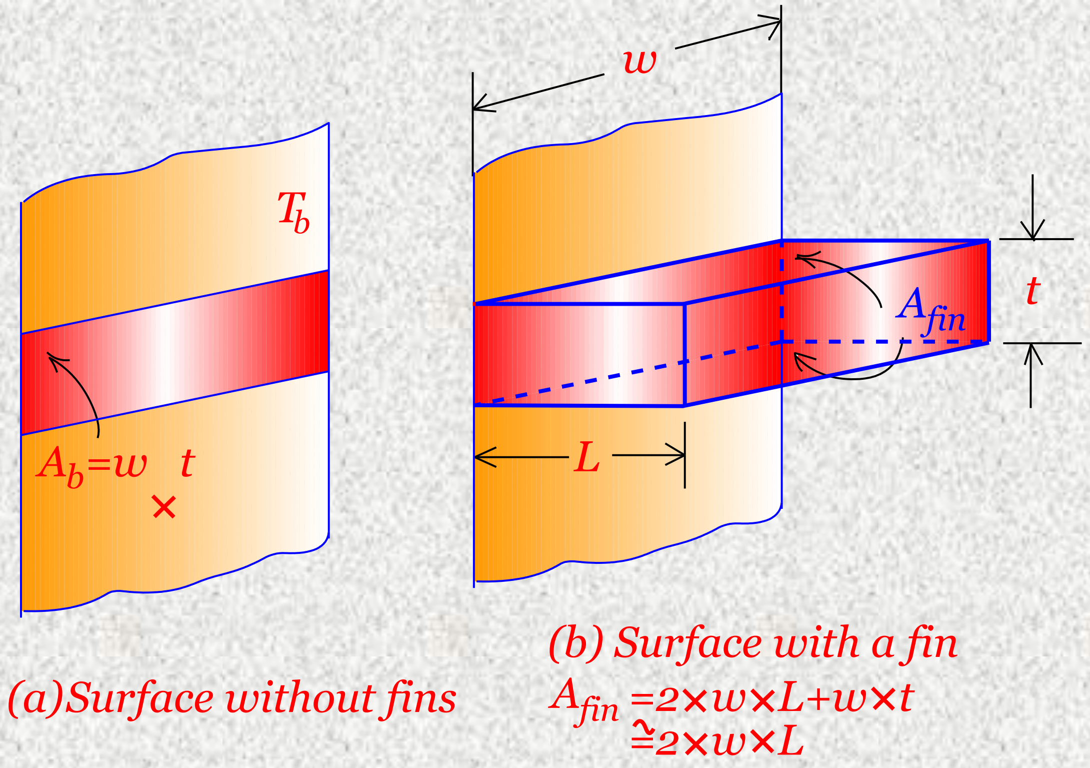


Figure : Fins Enhance Heat Transfer from a Surface by Enhancing Surface

- This time heat will flow from the surface to the fin by conduction.
- from the fin to the surrounding medium by convection with the same heat transfer coefficient *h.*

- The temperature of the fin will be T_b at the fin base and gradually decrease toward the fin tip.

- Convection from the fin surface causes the temperature at any cross section to drop somewhat from the midsection toward the outer surfaces.

- However, the cross sectional area of the fins is usually very small, and thus the temperature at any cross section can be considered to be uniform.

- Also, the fin tip can be assumed for convenience and simplicity to be insulated by using the corrected length for the fin instead of the actual length.

- In the limiting case of zero thermal resistance or infinite thermal conductivity ($k \to \infty$), the temperature of the fin will be uniform at the base value of T_b. The heat transfer from the fin will be maximum in this case and can be expressed as

$$Q_{fin,max} = h A_{fin} (T_b - T_\infty)$$

In reality, however, the temperature of the fin will drop along the fin, and thus the heat transfer from the fin will be less because of the decreasing temperature difference $T(x) - T_\infty$ toward the fin tip, as shown in Figure (on next slide)

To account for the effect of this decrease in temperature on heat transfer, we define fin efficiency as

$$\eta_{fin} = \frac{q_{fin}}{q_{fin,max}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if the entire fin were at base temperature}}$$

where A_{fin} is the total surface area of the fin.

This relation enables us to determine the heat transfer from a fin when its efficiency is known. For the cases of constant cross section of very long fins and fins with insulated tips, the fin efficiency can be expressed as

$$\eta_{long\ fin} = \frac{q_{fin}}{q_{fin,max}} = \frac{\sqrt{h P k A_c}\left(T_b - T_\infty\right)}{h A_{fin}\left(T_b - T_\infty\right)} = \frac{1}{L}\sqrt{\frac{h P}{k A_c}} = \frac{1}{m L}$$

and

$$\eta_{insulated} = \frac{q_{fin}}{q_{fin,max}} = \frac{\sqrt{h\,P\,k\,A_c}\left(T_b - T_\infty\right) Tanh\, mL}{h\,A_{fin}\left(T_b - T_\infty\right)}$$

$$\eta_{insulated} = \frac{q_{fin}}{q_{fin,max}} = \frac{TanhmL}{L}\sqrt{\frac{h\,P}{k\,A_c}} = \frac{Tanh\, mL}{m\,L}$$

Since A_{fin}=*PL* for fins with constant cross section. Above equation can also be used for fins subjected to convection provided that the fin length *L* is replaced by the corrected length L_c.

Fin efficiency relations are developed for fins of various profiles and are plotted in Figures for fins on a plain surface and for circular fins of constant thickness. The fin surface area associated with each profile is also given on each figure. For most fins of constant thickness encountered in practice, the fin thickness *t* is too small relative to the fin length *L* , and thus the fin tip area is negligible. Note that fins with triangular and parabolic profiles contain less material and are more efficient than the ones with rectangular profiles, and thus are more suitable for applications requiring minimum weight such as space applications.

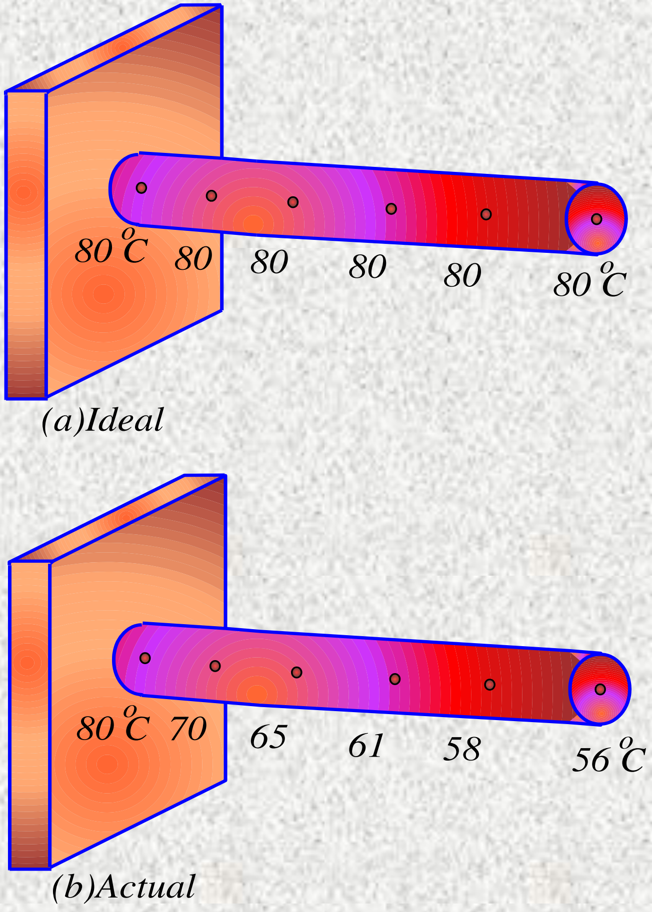


Figure 3.9 Ideal And Actual Temperature Distribution In A Fin

- An important consideration in the design of finned surfaces is the selection of the proper fin length *L*.
- Normally the longer the fin, the larger the heat transfer area and thus the higher the rate of heat transfer from the fin.
- But also the larger the fin, the bigger the mass, the higher the price, and the larger the fluid friction.
- Therefore, increasing the length of the fin beyond a certain value cannot be justified unless the added benefits outweigh the added cost.
- Also, the fin efficiency decreases with increasing fin length because of the decrease in fin temperature with length.
- Fin lengths that cause the fin efficiency to drop below 60 percent usually cannot be justified economically and should be avoided.
- The efficiency of most fins used in practice is above 90 percent.

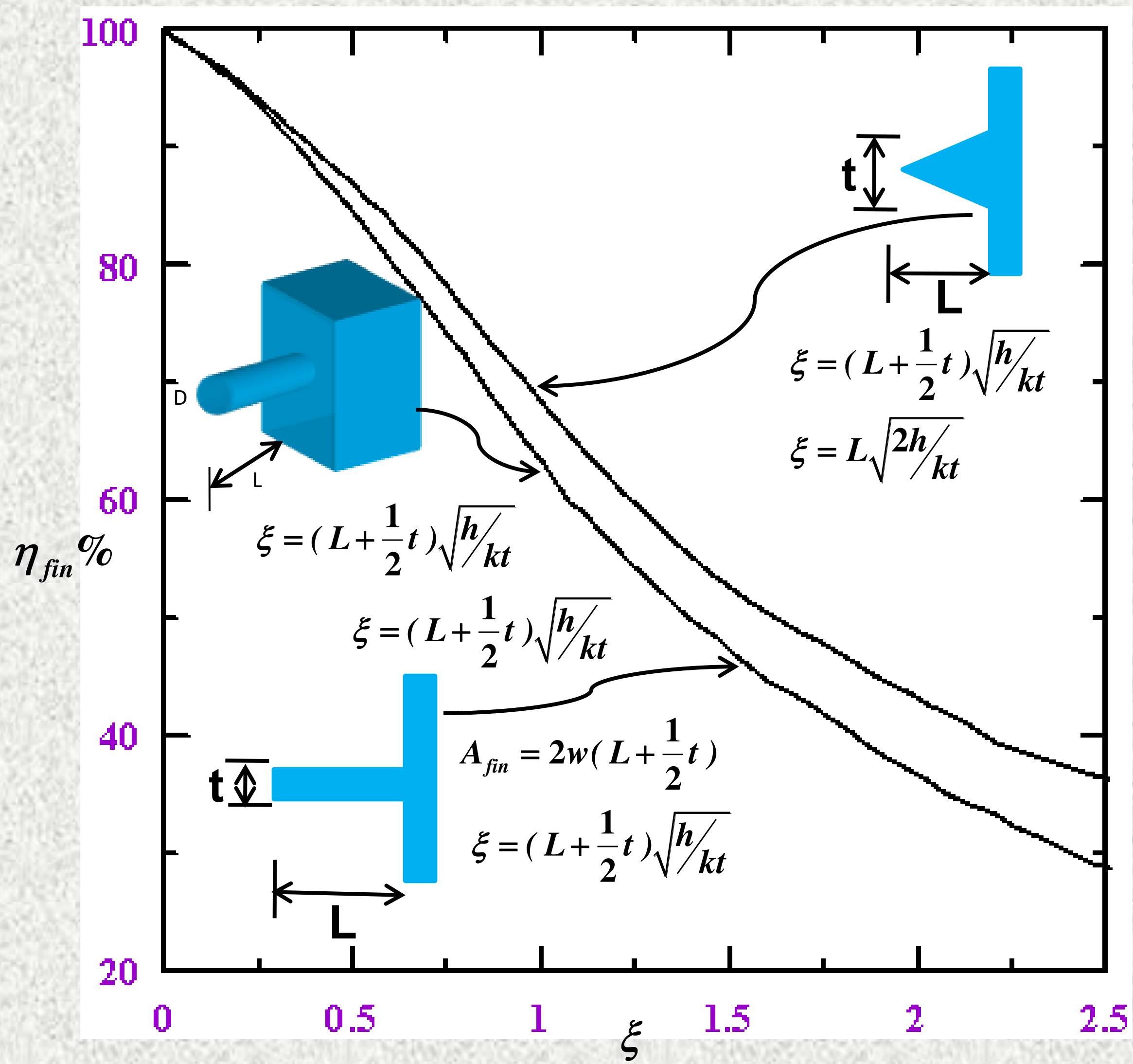


Figure: Efficiency Of Circular, Rectangular And Triangular Fins On A Plain Surface Of Width *w*

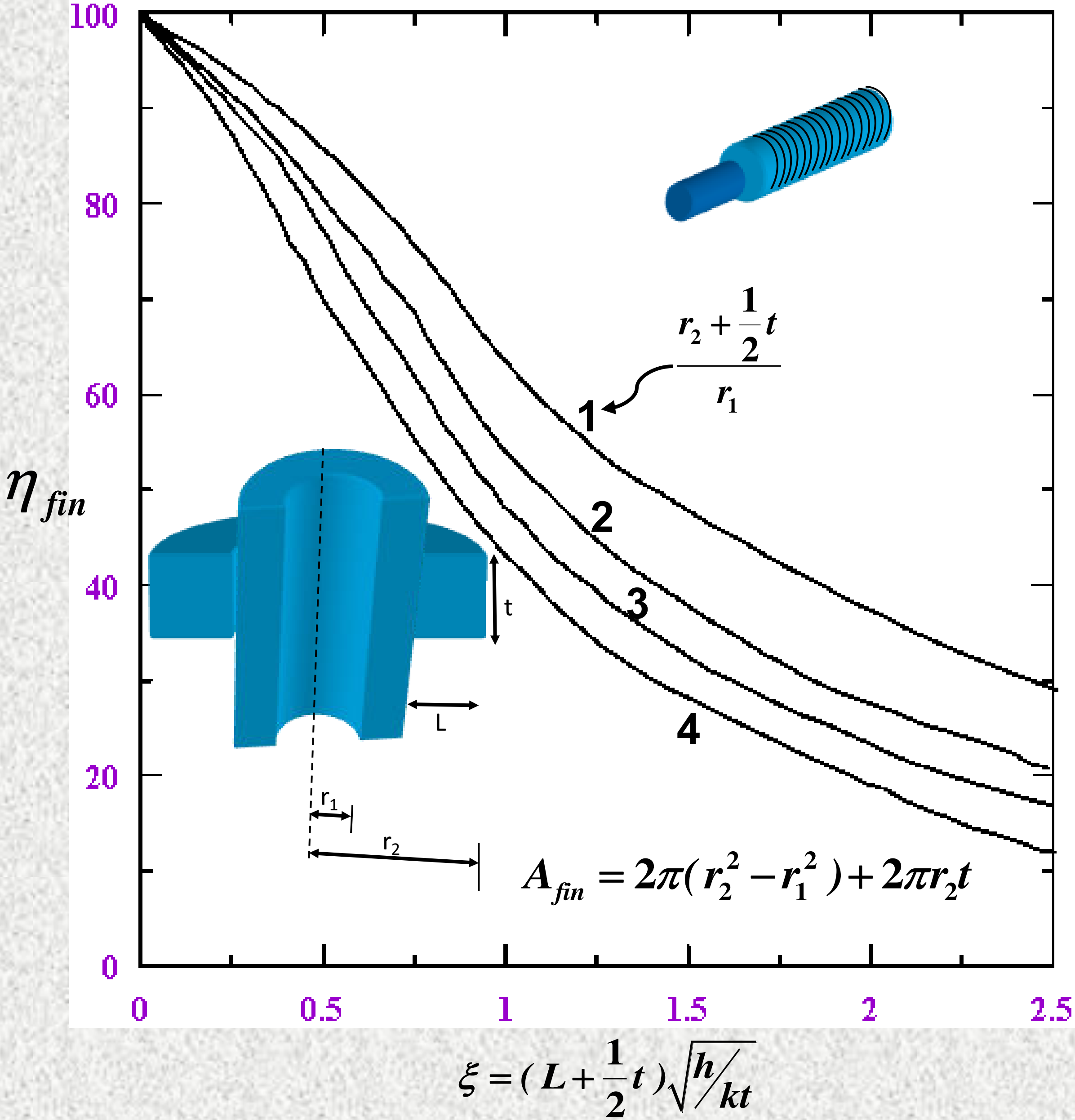


Figure: Efficiency Of Circular Fins Of Length *L* And Constant Thickness *T*

FIN EFFECTIVENESS

- Fins are used to enhance heat transfer, and the use of fins on a surface cannot be recommended unless the enhancement in heat transfer justifies the added cost and complexity associated with the fins.

- In fact, there is no assurance that adding fins on a surface will enhance heat transfer.

- The performance of the fins is judged on the basis of enhancement of heat transfer relative to the no fin case.

The performance of fins expressed in terms of the fin effectiveness ε_{fin} defined as (See figure)

$$\varepsilon_{fin} = \frac{q_{fin}}{h\,A_b\,(T_b - T_\infty)}$$

Here, A_b is the cross sectional area of the fin at the base and $q_{no\,fin}$ represents the rate of heat transfer from this area if no fins are attached to the surface.

The physical significance of effectiveness of fin can be summarized below

- An effectiveness of $\varepsilon_{fin} = 1$ indicates that the addition of fins to the surface does not affect heat transfer at all. That is, heat conducted to the fin through the base area A_b is equal to the heat transferred from the same area A_b to the surrounding medium

- An effectiveness of $\varepsilon_{fin} < 1$ indicates that the fin actually acts as insulation, slowing down the heat transfer from the surface. This situation can occur when fins made of low thermal conductivity materials are used.

- An effectiveness of $\varepsilon_{fin} > 1$ indicates that the fins are enhancing heat transfer from the surface, as they should. However, the use of fins cannot be justified unless ε_{fin} is sufficiently larger than 1. Finned surfaces are designed on the basis of maximizing effectiveness of a specified cost or minimizing cost for a desired effectiveness.

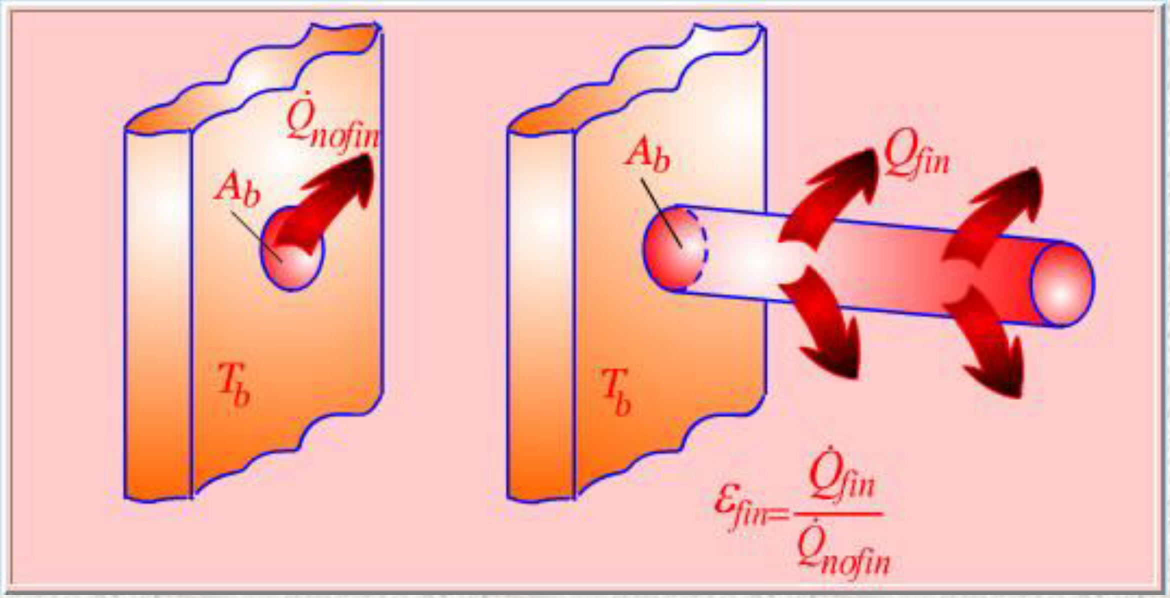


Figure: The Effectiveness Of The Fin

RELATION BETWEEN FIN EFFICIENCY AND FIN EFFECTIVENESS

The fin efficiency and fin effectiveness are related to the performance of the fin, but they are different quantities. However, they are related to each other by

$$\varepsilon_{fin}=\frac{q_{fin}}{q_{no\,fin}}=\frac{q_{fin}}{h\,A_b\,(T_b-T_\infty)}=\frac{\eta_{fin}\,h\,A_{fin}\,(T_b-T_\infty)}{h\,A_b\,(T_b-T_\infty)}=\frac{\eta_{fin}\,A_{fin}}{A_b}$$

Therefore, the fin effectiveness can be determined easily when the fin efficiency is known, or vice versa.

The rate of heat transfer from a sufficiently long fin or uniform cross section under steady conditions is given by Equation 3.34. Substituting this relation into Equation 3.40, the effectiveness of such a long fin is determined to be

$$\varepsilon_{fin} = \frac{q_{fin}}{q_{no\ fin}} = \frac{(T_b - T_\infty)\sqrt{h\,P\,k\,A_c}}{h\,A_b\,(T_b - T_\infty)} = \sqrt{\frac{kP}{hA_c}}$$

Since $A_c = A_b$ in this case. We can draw several important conclusions from the fin effectiveness relation above for consideration in the design and selection of the fins

- The thermal conductivity *k* of the fin material should be as high as possible. Thus it is no coincidence that fins are made from metals, with copper, aluminum, and iron being the most common ones. Perhaps the most widely used fins are made of aluminum because of its low cost and weight and its resistance to corrosion.

- The ratio of the perimeter to the cross sectional area of the fin P/A_c should be as high as possible. This criterion is satisfied by thin plate fins or slender pin fins

- The use of fins is most effective in applications involving low convection heat transfer coefficient. Thus, the use of fins is more easily justified when the medium is a gas instead of a liquid and the heat transfer is by natural convection instead of by forced convection. Therefore, it is no coincidence that in liquid-to-gas heat exchangers such as the car radiator, fins are placed on the gas side.

When determining the rate of heat transfer from a finned surface, we must consider the unfinned portion of the surface as well as the fins. Therefore, the rate of heat transfer for a surface containing *n* fins can be expressed as

$$q_{total,fin} = q_{unfin} + q_{fin} = h A_{unfin}(T_b - T_\infty) + \eta_{fin} A_{fin}(T_b - T_\infty)$$

$$q_{total,fin} = h(A_{unfin} + \eta_{fin} A_{fin})(T_b - T_\infty)$$

We can also define an overall effectiveness for a finned surface as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins,

$$\varepsilon_{fin} = \frac{q_{fin}}{q_{no\ fin}} = \frac{h\left(A_{unfin} + \eta_{fin} A_{fin}\right)\left(T_b - T_\infty\right)}{h\, A_{no\ fin}\left(T_b - T_\infty\right)}$$

where

- A_{nofin} is the area of the surface when there are no fins,
- A_{fin} is the total surface area of all the fins on the surface, and
- A_{unfin} is the area of the unfinned portion of the surface (Figure: next slide_.

Note that the overall fin effectiveness depends on the fin density (i.e. number of fins per unit length) as well as the effectiveness of the individual fins. The overall effectiveness is a better measure of the performance of a finned surface than the effectiveness of the individual fins.

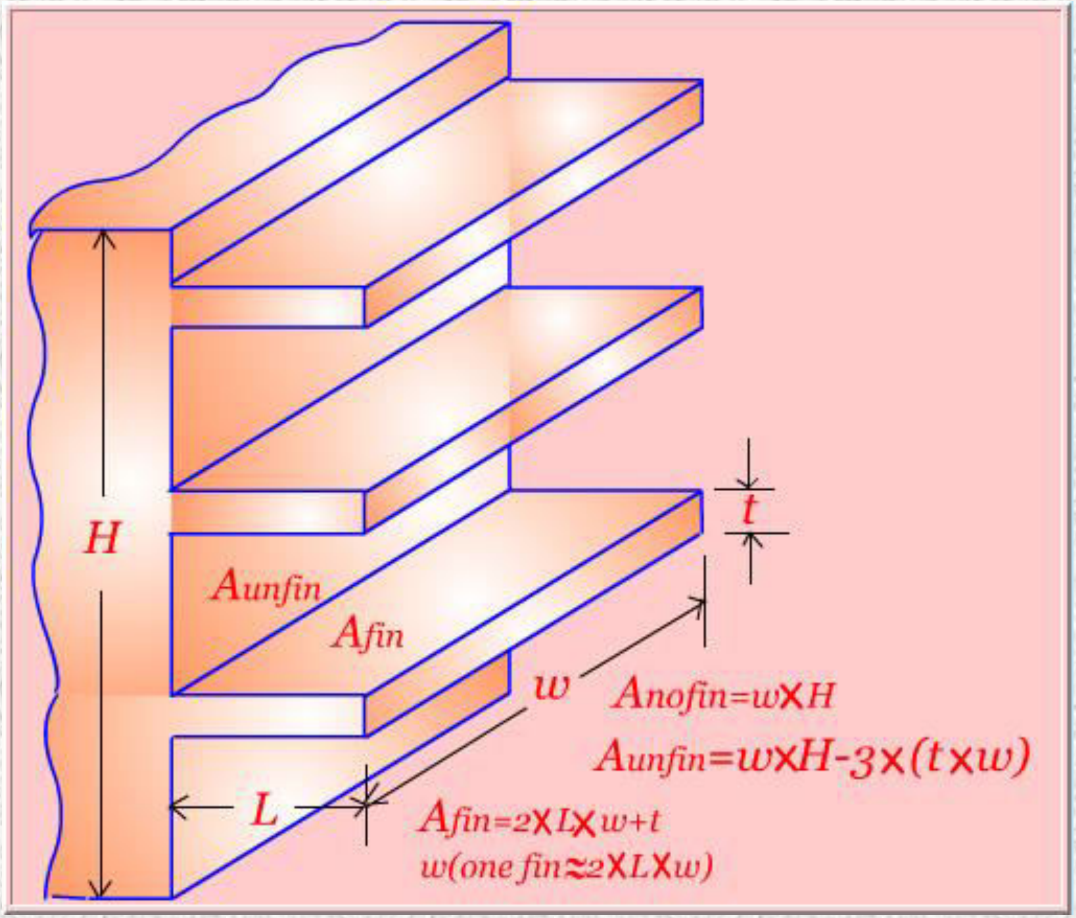


Figure Various Surface Areas Associated With A Rectangular Surface With Three Fins

PROPER LENGTH OF THE FIN

- An important step in the design of a fin is the determination of the appropriate length of the fin once the fin material and the fin cross section are specified.

- You may be tempted to think that the longer the fin, the larger the surface area and thus the higher the rate of heat transfer. Therefore, for maximum heat transfer, the fin should be infinitely long.

- However, the temperature drops along the fin exponentially and reaches the environment temperature at some length.

- The part of the fin beyond this length does not contribute to heat transfer since it is at the temperature of the environment, as shown in Figure.

- Therefore, designing such an “extra long” fin is out of question since it results in material waste, excessive weight, and increased size and thus increased cost with no benefit in return (in fact, such a long fin will hurt performance since it will suppress fluid motion and thus reduce the convection heat transfer coefficient).

Therefore, fins that are so long that the temperature approaches the environment temperature cannot be recommended

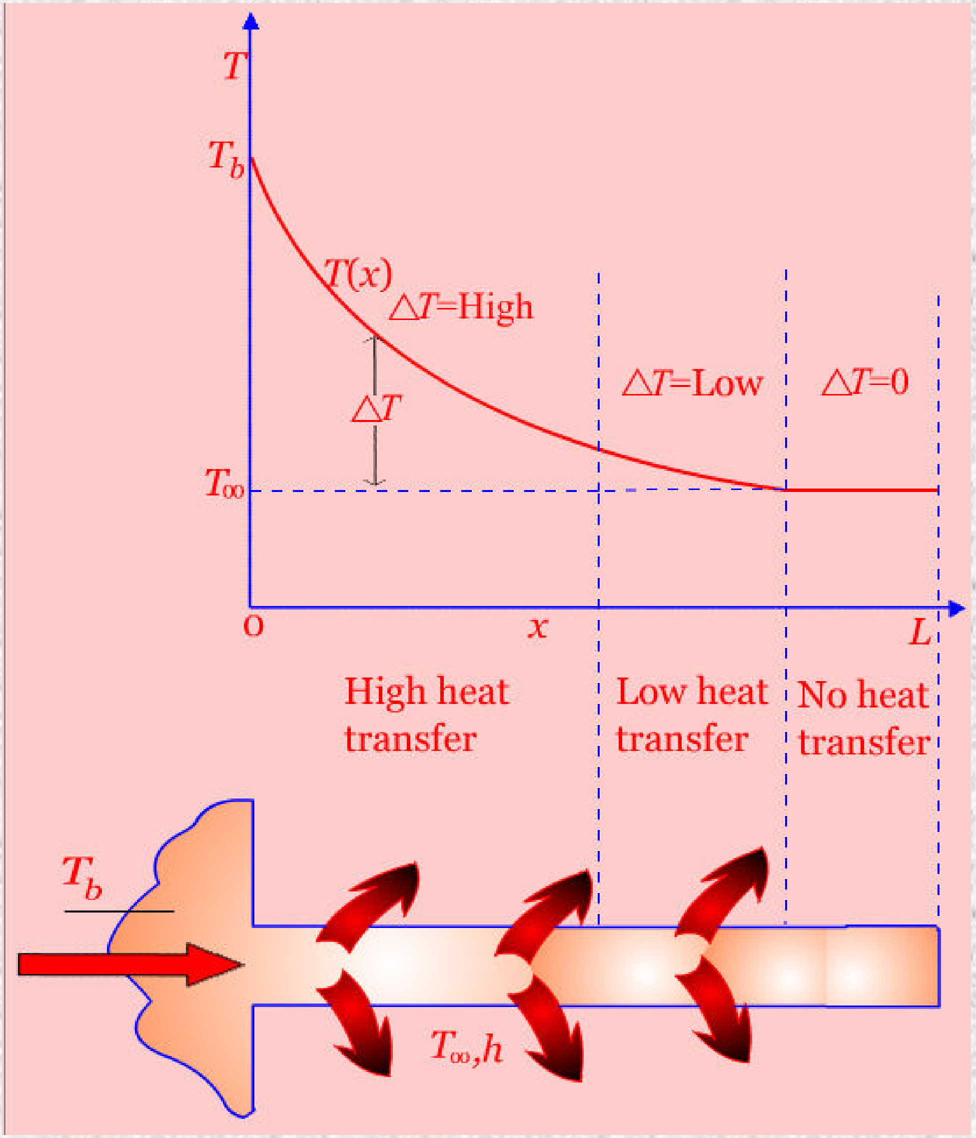


Figure Temperature Drop along the Fin

To get a sense of proper length of a fin, we compare heat transfer from a fin of finite length to heat transfer from an infinitely long fins under the same conditions. The ratio of these two heat transfers is heat transfer ratio

$$\frac{q_{fin}}{q_{no\ fin}} = \frac{\sqrt{h P k A_c}\,(T_b - T_\infty)\,Tanh\,mL}{\sqrt{h P k A_c}\,(T_b - T_\infty)} = Tanh\,mL$$

The values of *Tanh mL* are evaluated for some values of *mL* and the results are given in Table.

Table: The variation of heat transfer from a fin relative to that from an infinitely long fin

mL	$Tanh mL$
0.1	0.1
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

- We observe from the table that heat transfer from a fin increases with *mL* almost linearly at first, but the curve reaches a plateau later and reaches a value for the infinitely long fin at about *mL=5*.

- Therefore, a fin whose length is *L=m/5* can be considered to be an infinitely long fin.

- We also observe that reducing the fin length by half in that case (from *mL=5* to *mL=2.5*) causes a drop of just 1 percent in heat transfer.
- We certainly would not hesitate sacrificing 1 percent in heat transfer performance in return for 50 percent reduction in the size and possibly the cost of the fin.
- In practice, a fin length that corresponds to about *mL=1* will transfer 76.2 percent of the heat that can be transferred by an infinitely long fin, and thus it should offer a good compromise between heat transfer performance and the fin size.

Optimization of fin thickness

$$\varepsilon = \text{Effectiveness} = \frac{q_f}{hA_{c,b}\theta_b}$$

For infinitely long fin approximate ε is given as

$$\varepsilon = \sqrt{\frac{kP}{hA}} \qquad k\uparrow \quad \mathrm{P}\uparrow \quad \mathrm{h}\downarrow \quad \mathrm{A}\downarrow$$

Assume that the fin volume is fixed and try to maximize the heat transfer from the base

$$Volume = LtW$$

Assume that the fin width is known

$$Q = \sqrt{hPkA}\,\theta_b \tanh ml$$

Optimization of fin thickness

$$P = 2W + 2t \approx 2w; \quad A = Wt$$

$$\sqrt{hPkA} = \sqrt{h(2W)kWt} = W\sqrt{2hk}\sqrt{t}$$

$$mL = \sqrt{\frac{hP}{kA}}L = \sqrt{\frac{h2W}{kWt}}\frac{V}{Wt} = \sqrt{\frac{2h}{k}}\frac{V}{W}t^{-3/2}$$

$$Q = \theta_b W\sqrt{2hk}\sqrt{t}\tanh\sqrt{\frac{2h}{k}}\frac{V}{W}t^{-3/2}$$

$$Q = a\sqrt{t}\tanh bt^{-3/2}$$

To optimize thickness use: $\frac{\partial Q}{\partial t} = 0$

Optimization of fin thickness

$$a\tanh\left(bt^{-3/2}\right)\frac{1}{2}t^{-1/2}+a\sqrt{t}\left\{1-\tan^2 h(bt^{-3/2})\right\}\left(\frac{3}{2}bt^{-5/2}\right)=0$$

$$\frac{1}{2}\tanh\left(bt^{-3/2}\right)+\left\{1-\tan^2 h(bt^{-3/2})\right\}\left(-\frac{3}{2}b\right)t^{-3/2}=0$$

Let $bt^{-3/2}=A$

$$\frac{1}{2}\tanh A+\left\{1-\tan^2 hA)\right\}\left(-\frac{3}{2}A\right)=0$$

Solving for A gives $A=1.42$

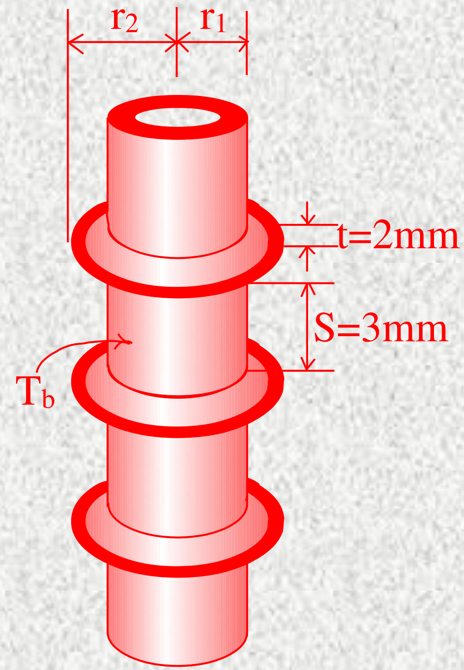
$$t\approx\left(\frac{V}{W}\right)^{2/3}\left(\frac{h}{k}\right)^{1/3}$$

$$L\approx\left(\frac{V}{W}\right)^{1/3}\left(\frac{h}{k}\right)^{-1/3}$$

Problem 3.2:

Steam in a heating system flows through tubes whose outer diameter is D_1= 3 cm and whose walls are maintained at a temperature of 125°C. Circular aluminium fins (k = 180 W/m°C) of outer diameter D_2= 6 cm and constant thickness t = 2mm are attached to the tube, as shown in the Figure. The space between the fins is 3 mm, and thus there are 200 fins per meter length of the tube. Heat is transferred to the surrounding air at T_∞ = 27°C, with a combined heat transfer coefficient of h = 60 W/m^2 °C. Determine the increase in heat transfer from the tube per meter of its length as a result of adding fins.

Figure



Known: Properties of the fin, ambient conditions, heat transfer coefficient, dimensions of the fin.

Find: To find the increase in heat transfer from the tube per meter of its length as a result of adding fins.

Assumptions:

1. Steady operating conditions exist.
2. The heat transfer coefficient is uniform over the entire fin surfaces.
3. Thermal conductivity is constant.
4. Heat transfer by radiation is negligible.

Analysis:

In the case of no fins, heat transfer from the tube per meter of its length is determined from Newton's law of cooling to be,

$$A_{no\ fin} = \pi D_1 L = \pi (0.03)(1) = 0.0942\, m^2$$

$$\dot{Q}_{no\ fin} = h A_{no\ fin} (T_b - T_\infty)$$

$$\dot{Q}_{no\ fin} = 60 \times 0.0942(125-27) = 554\,\mathrm{W}$$

The efficiency of the circular fins attached to a circular tube is plotted in Figure 3.21. Noting that $L = ½(D_2 – D_1)$ = ½ (0.06 – 0.03) = 0.015m in this case, we have

$$\frac{r_2 + \frac{1}{2}t}{r_1} = \frac{0.003 + \left(\frac{1}{2}\right)0.002}{0.015} = 0.27$$

$$\left(L + \frac{1}{2}t\right)\sqrt{\frac{h}{k\,t}} = \left(0.015 + \frac{1}{2}(0.002)\right)\sqrt{\frac{60}{(180)(0.002)}} = 0.21$$

Hence, η_{fin}= 0.95.

$$A_{fin} = 2\pi\left(r_2^2 - r_1^2\right) + 2\pi\, r_2 t$$

$$A_{fin} = 2\pi\left((0.03)^2 - (0.015)^2\right) + 2\pi(0.03)(0.002)$$

$$A_{fin} = 0.00462\ m^2$$

$$\dot{Q}_{fin} = \eta_{fin}\,\dot{Q}_{fin\,max} = \eta_{fin}\,h\,A_{fin}\left(T_b - T_\infty\right)$$

$$\dot{Q}_{fin} = 0.95(60)(0.00462)(125-27) = 25.8\,\mathrm{W}$$

Noting that the space between the two fins is 3 mm, heat transfer from the unfinned portion of the tube is

$$A_{unfin} = \pi D_1 S = \pi(0.03)(0.003) = 0.000283\ \text{m}^2$$

$$\dot{Q}_{unfin} = h A_{unfin}(T_b - T_\infty) = 60(0.000283)(125-27) = 1.66\,\text{W}$$

Noting that there are 200 fins and thus 200 inter-fin spacings per meter length of the tube, the total heat transfer from the finned tube becomes

$$\dot{Q}_{total,\,fin} = n\left(\dot{Q}_{fin} + \dot{Q}_{unfin}\right) = 200(25.8+1.66) = 5492\,\text{W}$$

Therefore, the increase in the heat transfer from the tube per meter of its length as a result of the addition of fins is

$$\dot{Q}_{increase} = \dot{Q}_{total,\,fin} - \dot{Q}_{no\ fin} = 5492 - 554 = 4938\,\text{W}\ (\text{per m of tube length})$$

Comments:

The overall effectiveness of the finned tube is

$$\varepsilon_{fin,overall} = \frac{\dot{Q}_{total,\,fin}}{\dot{Q}_{total,\,no\ fin}} = \frac{5492}{554} = 9.91$$

That is, the rate of heat transfer from the steam tube increases by a factor of almost 10 as a result of adding fins. This explains the widespread use of the finned surface.